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Single Item Periodic Review Inventory Control with Sales Dependent Stochastic Return Flows

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Abstract Retailers have to deal with increasing levels of product returns as the shares of e-commerce sales soars. With this increase, it is no longer feasible to dispatch returned products to outlets or landfills, hence retailers must re-evaluate them both to maximize profit and to minimize their environmental impact. Our objective is to study a retailer's optimal inventory control policy under product returns to maximize expected profit which is the sales revenue minus the procurement, backorder, holding, and salvage costs incurred in a finite horizon. We model a period's returns to be stochastically dependent on the previous period's sales quantity. Using dynamic programming formulation, we solve for the optimal periodic review inventory policy and provide structural results on the optimal policy of the final period. Through numerical studies, we show that incorporating detailed sales-dependent returns could increase a retailer's expected profit by 23%. Ignoring this dependency in determining the optimal inventory policy results with increased order frequency, higher levels of backorders and more leftovers which could eventually end up in a landfill, but above all could lead to a significant overestimation of the resulting profit.

Keywords inventory management · stochastic product returns · dynamic programming

1 Introduction

The retail industry is gigantic and was worth nearly $24 trillion in 2019 (O'Connell 2020). The industry is mostly run by large retail chains since they have the cost advantage of buying huge amounts of inventory. Walmart, the largest retail chain in the

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world, is top-ranked among all industries in Fortune Global 500 list for years. As the retail industry expands and evolves with digitalization, consumers with dynamic purchasing behaviors are also pushing for higher levels of personalization and quality. To overcome this consumer hunger, retailers increase the number of products they offer. For example, Walmart offers more than 75 million stock keeping units for sale on its online channel (Lore 2018).

A majority of the retail chains sell through both off-line and online stores, with an increasing focus on the latter, and allow customers to experience a smooth service from both channels. The Chinese company, Alibaba Group, significantly increased the online retail sales with its 580 million active monthly users while European or US countries face the brick and mortar store closures (Devani and Coonan 2018). During the Covid-19 pandemic, the retail industry has even struggled with jumping online sales due to "stay at home" obligations. Bhattarai (2020) states that Walmart's online sales increased by 74%, lifting overall sales by nearly 9% in two months. Adobe's Digital Economy Index for July 2020 reveals that US online sales increased 55% from last year, despite a slowdown in the growth due to reopened states (AdobeAnalytics 2020). According to the report, online shopping was preferred for cheaper prices before the pandemic but now it seems that this differentiation is no longer valid and in fact, online sales are substituting off-line sales.

As retail e-commerce grows rapidly, product returns are also growing. Either for legislative or competitive reasons, many companies accept returns without any reason declared, under the "No Questions Asked Return Policy" (Ülkü and Gürler 2018), and this customer experience increases sales. The National Retail Federation states that $260 billion in merchandise has been returned in 2016 in the US, which is a 66% increase from five years ago. The lack of power of touch at the online stores further increases the number of returns. Smith (2015) presents that a brick-and-mortar store faces returns as the 10% of the sales, whereas this ratio is 20% with e-commerce returns. Holiday e-commerce returns increase to 30% of the sales and even to 50% of the sales for expensive products. Increasing return rates are also attributable to strategic customers who are abusing the return policies of retail companies and ordering more than they need to resolve fit uncertainty and then simply returning the unfit fashion items (Ülkü and Gürler 2018). Mostard and Teunter (2006) reports that for catalog retailers return rates on fashion items are generally around 35–40% and could increase to 75% of the sales for some products.

Product returns are often accepted for longer periods when there is a defect under product warranty. This return process is often protected by law. Companies also may accept returns of no-defect products for a shorter declared period even if it is not always obligatory, which is the focus of this paper. Retailers can increase consumer satisfaction and loyalty and collect data regarding consumer behavior or product performance by allowing returns of no-defect products. Retailers also have the advantage of using the funds obtained from the revenue of the returned products till the product is eventually returned and the customer is refunded, which could take a substantial amount of time. However, handling product returns could be costly especially when the returns are not resold. If the returns are not added back to the inventory in the selling season, they could stay in the warehouse generating holding costs until they are resold or salvaged or could go to landfill generating obsolescence or environmental costs. Hence retailers are in a position to carefully manage their inventories to address these challenges in order to reap the benefits of product returns.

Inventory management under product returns requires detailed sales and returns data. Trivially, the number of product returns depends on the previous product sales: as sales increases, returns in the following period are also likely to increase. However, relevant data to measure this dependency may not be available. Not all companies keep a separate record of sales and return data due to practical reasons and instead keep track of net sales which is product sales minus returns in a particular period. When the return rates are low, the net sales approach is convenient. In the retail sector, some firms, especially those with e-commerce channels, keep very detailed sales and returns data. In this paper, we will present an inventory control model in which stochastic returns depend on previous sales. Our model would help firms to quantify the benefits of keeping detailed sales and returns data, and how to leverage this data in managing product inventories.

In the literature, there are many papers on inventory management of remanufactured or recycled return items (see, for example, Fleischmann and Kuik 2003, Benedito and Corominas 2013. Another stream of literature focuses on return items that can be sold immediately without any processing (see, for example, Kiesmüller and Van der Laan 2001 and Zerhouni et al. 2013. We contribute to the latter stream of literature by incorporating the dependency of returns on previous sales. In our model, we maximize a retailer's expected total revenue less the costs of fixed order, procurement, backorder, holding, and salvage incurred in a finite horizon. We assume that product returns are stochastically dependent on the previous period's sales quantity.

Our main contributions to the existing literature can be summarized as follows: (1) The form of stochastic dependency of returns on the previous sales modelled in this paper is original and rich in details and thus operationally more relevant for the practitioners. (2) Using dynamic programming, we derive an optimal periodic review inventory policy for our problem and demonstrate that an (s,S) type inventory policy may no longer be optimal. (3) We show some structural properties of the optimal inventory policy in the last period.(4) We compare the profits of a *return-smart* retailer who manages its inventory by keeping separate records of sales and returns and following our results and a *return-naive* retailer who does not track sales explicitly and simply uses the net demand approach (i.e., considers only the product demand less the product returns). Using Monte Carlo simulation, we find that the return-smart retailer could enjoy on average 23% more profit than the return-naive retailer. We find that this profit improvement percentage is most sensitive to the fixed order cost, backorder cost, and return rates. The return-smart retailer considers the possible incoming product returns and orders less frequently (thus saves from the fixed order cost) compared to the return-naive retailer. The latter option leads to lower order-up-to levels which increase backorders as the return rates increase. (5) Finally, we provide an extension of our model with backorders to the case with lost sales.

The paper is organized further as follows. In Section 2, we review the related literature. In Section 3, we describe the details of the model and our assumptions. Analytical results on the formulated problem are discussed in Section 4. We exhibit the results of a numerical study along with a sensitivity analysis in the next section.

We present an extension with the lost sales case in Section 6. Finally, we summarize the results and conclusions and give an outlook for future research in Section 7.

2 Literature Review

Product returns have been studied in the literature along several lines of research such as inventory management with return flow (see, for example, Reimann 2016), the effects of return cost (see, for example, Shulman et al. 2010), and pricing strategy under product returns with money-back guarantee policy (see, for example, Chen et al. 2019). We focus on the inventory management implications of product returns, which have been studied in literature since the late seventies. The first papers are about disposal, remanufacturing, or recycling decisions for product returns. In this section, we review papers that use stochastic models and refer the readers on papers with deterministic models to Schrady (1967) as one of the earlier works and Fleischmann et al. (1997) for various extensions. We consider two streams for the papers in this group: The papers in the first stream assume that returns are not directly added back to inventory upon receipt but either remanufactured or disposed of, if there is no need for new products. In the second stream, papers assume that all returns are directly added back to inventory to satisfy future demand, similar to the setting in this paper.

The first stream includes papers focusing on hybrid manufacturing, remanufacturing, and disposal processes. Demand and return distributions are mostly assumed to be independent across periods and a net demand approach is followed which simply uses the demand amount after the returns are deducted. Only a handful of these papers assume that demand and returns are correlated albeit within the same period. For example, Simpson (1978) and Inderfurth (1997) study settings where a customer will need a brand new product upon return of an old product that reached its end of life. The papers in this stream could be further grouped based on the inventory review period employed.

One of the papers that studies a continuous review of the optimal disposal policy with returns is Heyman (1977). The paper aims to determine how many of the returns will be remanufactured and how many of them will be disposed of. Heyman (1978) extends this previous model and instead of disposal, excess returns are assumed to be sent to a central warehouse. In his model when the inventory of returned items reaches a certain value, then inventory is decreased to a specific level by initiating a remanufacturing cycle. Muckstadt and Isaac (1981) extend the preceding study by adding lead time and fixed order cost under both single and two-echelon systems. Van der Laan et al. (1996) extends the model studied in the preceding study with the possibility of partial disposition. Fleischmann et al. (2002) revises Muckstadt and Isaac (1981)'s model by eliminating remanufacturing requirement and allowing returns to be added back to inventory upon receipt. They find that the conventional (s, Q) inventory policy is still optimal under return flow. A multi-echelon extension to the continuous review inventory control with product returns problem is studied in Mitra (2009). Flapper et al. (2012) studies an infinite horizon continous review model with returns where demand and returns are independent. Gayon et al. (2017) studies a similar model with an extension of manufacturing fixed cost and lead times.

The papers in the second group consider periodic review inventory policies. Simpson (1978) finds the optimal solution to an *n*-period repairable return inventory problem. Inderfurth (1997) incorporates positive lead times to Simpson (1978)'s model. Kiesmüller and Scherer (2003) suggests effective heuristics to solve the models presented in the preceding two papers. Fleischmann and Kuik (2003) studies the average cost optimality of an (s, S) manufacturing policy with return flow using net demand in an infinite horizon assuming dependence of stochastic demand and returns within the same period. This work is extended in Mahadevan et al. (2003) with a push policy that controls the release time of the returned products to the remanufacturing line and decides the manufacturing quantity of new products. DeCroix (2006) extends the first two papers in this group via a serial multi-echelon inventory system with return flow in a finite horizon. They find the optimal inventory policy when re-manufactured items flow into the most upstream stage. Mitra (2013) extends this study with correlated demand and returns within the same period. Calmon and Graves (2017) incorporates fixed cost and lead time setting to Simpson (1978) but assumes that unmet demand is outsourced from an external source instead of a backorder model. More recently Fu et al. (2019) studies a model that assumes stochastic dependency of the returns and sales within the same period.

Papers in the second stream studies inventory control problems under recycling of returns. The recycled (i.e., returned) items are mostly added back to inventory without further processing (e.g., merchandise, containers, blood) or sometimes requires some processing but lead time or cost involved in such operations is ignored in most of the models. Since this group includes many papers, we simply concentrate on the papers in which demand and returns are assumed to be dependent across periods.

Cohen et al. (1980) studies a periodic review inventory system with recycling in which returns are deterministic and modeled to be a fixed fraction of demand. The paper assumes lost sales in a finite horizon with no fixed order cost or lead time. Kelle and Silver (1989) study a similar model with stochastic returns. Using simulation, the authors compare different return forecasting methods based on the granularity of the return and demand data (e.g., only aggregate, past period(s) demand, past period(s) return) and find the reorder point given the return forecast. Buchanan and Abad (1998) employs dynamic programming to solve the similar model of the preceding work assuming finite horizon. Yuan and Cheung (1998) studies a continuous review (s, S) inventory system for rental products in which stochastic returns are dependent on demand. In a finite period model, Kiesmüller and Van der Laan (2001) studies a periodic review inventory system where product returns are dependent on demand under positive total lead time. Zerhouni et al. (2013) uses Yuan and Cheung (1998)'s model and compares the demand-dependent return model with an independent one and suggest a heuristic to solve the problem. Benedito and Corominas (2013) considers a similar setting of Kelle and Silver (1989) and Kiesmüller and Van der Laan (2001) where the amount of returns depends on the useful life of the products sold and the probability of returns.

Table 1 provides a taxonomy of the most relevant papers to this work. In the first column, we group the papers based on the studied inventory control method e.g., pe-

Table 1: Detailed comparison of closely related papers

	Review Type	Return Dependency	Solution Method	Unmet Demand	Horizon	Fixed Cost	Lead Time	Demand Distribution	Return Distribution
Heyman (1977)	C	I	MC	NA	F			Poisson	Poisson
Heyman (1978)	C	I	MC	NA	IF			Poisson	Poisson
Simpson (1978)	P	DWP	DP	BO	F			Generic	Generic
Muckstadt and Isaac (1981)	C	I	MC	BO	IF	*	*	Poisson	Poisson
Kelle and Silver (1989)	P	DAP	S	NA	F		*	Normal	Binomial
van der Laan et al.(1996)	C	I	MC/H	BO	IF	*	*	Poisson	Poisson
Inderfurth (1997)	P	DWP	DP	BO	F		*	Generic	Generic
Yuan and Cheung (1998)	C	DAP	MC/H	BO	IF	*		Poisson	Exponential
Buchanan and Abad (1998)	P	DAP	DP	BO	F			Generic (Exp.)	Generic (Uniform)
Kiesmüller and van der Laan (2001)	P	DAP	MC	BO	F		*	Poisson	Poisson
Fleischmann et al. (2002)	C	I	MC	BO	IF	*	*	Poisson	Poisson
Fleischmann and Kuik (2003)	P	DWP	MC	BO	IF	*	*	Generic (Poisson)	Generic (Poisson)
Kiesmüller and Scherer (2003)	P	I	DP/H	BO	F		*	Normal	Normal
Mahadevan et al. (2003)	P	I	H/S	BO	IF		*	Poisson	Poisson
De croix (2006)	P	DWP	DP	BO	F		*	Generic	Generic
Mitra (2009)	C	I	S	BO	IF	*	*	Normal	Normal
Flapper et al. (2012)	C	I	MC	LS	IF			Poisson	Poisson
Mitra (2013)	P	DWP	S	BO	IF	*		Normal	Normal
Benedito and Corominas (2013)	P	DAP	MC/H	NA	IF			Generic	Generic (Binomial)
Zerhouni et al. (2013)	C	DAP	MC/H	LS	IF			Poisson	Exponential
Calmon and Graves (2017)	P	DWP	S/H	NA	F	*	*	Generic	Generic
Gayon et al. (2017)	C	I	MC	BO	IF	*	*	Poisson	Poisson
Fu et al.(2019)	P	SWP	DP	LS	F			Generic (Uniform)	Generic (Uniform)
Our Paper	P	SAP	DP	BO	F	*		Generic (Poisson)	Binomial

riodic review (P) and continuous review (C). The next column specifies the return dependency on demand under five clusters: independent from demand (I), demand dependent within the same period (DWP), demand dependent across periods (DAP), sales-dependent within the same period (SWP), and sales-dependent across periods (SAP). In the third column, we indicate the employed solution methodology: dynamic programming (DP), Markov Chain modeling (MC), simulation (S), or heuristics (H). We also label the unmet demand assumption as backorder (BO), lost sales (LS) or an instantaneous outsource supply (NA) in column four. The next one lists whether the model covers a finite (F) or an infinite horizon (IF). Columns six and seven denotes papers with fixed ordering cost or lead times, respectively, for procurement or remanufacturing with an asterisk. The last two columns specify the distributions used to model demand and return in the main model and the computational study in parenthesis.

In this paper, we employ dynamic programming to solve for the optimal inventory control policy of a firm that accepts returns that depend on previous period sales. Moreover, we quantify the benefit of using this detailed approach instead of ignoring the dependency between sales and returns. Our contribution to the literature is to present and study a model with previous sales-dependent stochastic return flows while including fixed ordering costs in a finite horizon setting. We believe the most significant contribution is to model the dependency between previous period sales and the next period returns. The stochastic dependency of returns on the previous sales as assumed in Buchanan and Abad (1998) and Benedito and Corominas (2013) translates into a dependency of returns and demand, as any unmet demand can be satisfied instantaneously in these papers. In a two-period model for a perishable product, Fu et al. (2019) assumes that returns are dependent only on the current period's sales. Another contribution of this paper is to include a fixed ordering cost, which has not been studied previously in conjunction with return dependency. These additional considerations result in a detailed model and thus practically relevant conclusions about managing inventories under return flows.

3 Model Formulation

We consider a retailer's finite horizon periodic review inventory control problem where the product returns depend on the previous period sales quantity. We assume that the retailer has a single store and focus on a single item that is allowed to be returned within one period upon purchase without any restrictions. The returned product is assumed to be in almost perfect condition and is directly put on the shelf for resale as a new product upon arrival to the store after minor processing which is assumed to have negligible cost and takes negligible time.

The retailer reviews its net inventory at the beginning of each period t, I_t and gives a replenishment order, O_t, if necessary, which is received after a negligible replenishment lead time. Next, product returns, R_t, are accepted and then demand (D_t) and resulting sales (SL_t) are realized. Any unsatisfied demand is backordered (B_t). At the end of the period, holding or backorder costs are incurred. We assume that the selling season for this product ends after T periods. After the end of the selling season, either the final shortage is satisfied via a final procurement or the remaining inventory is salvaged. A list of notations is provided in Table 2.

Table 2: List of notations used in this paper

c	Unit procurement cost
K	Fixed ordering cost
h	Holding cost per unit per period
b	Backorder cost per unit per period
r	Return credit
s	Salvage value
p	Retail price
α	Return probability
T	Number of periods in the selling season
I_t	Net inventory at the beginning of period t
O_t	Replenishment order in period t
D_t	Demand realized during period t
SL_t	Sales realized during period t
R_t	Returns received during period t
B_t	Backorder quantity at the end of period t
S_t	Inventory level just after replenishment order but before demand and returns

We assume that the demand across periods, D_t, is independent and identically distributed and is a discrete random variable with probability mass function $f(x)$ and mean μ. Customers pay a unit price of p to purchase the product. Each sold product in period t is assumed to be returned in period $t+1$ with probability α. We assume that return probability is independent of the sales period, but our model can be extended to incorporate nonidentical return probabilities across sales periods. Given the previous period sales SL_{t-1}, let $CR_{i,t}$ denote a Bernoulli random variable for sold product i's ($i \in \{1,2,...,SL_{t-1}\}$) return status ($CR_{i,t} = 1$ implies a returned product). One could then calculate the total return quantity in period t as follows:

$$R_t = \sum_{i=1}^{SL_{t-1}} CR_{i,t}$$

which trivially follows a binomial distribution with parameters α and SL_{t-1}. Immediately upon the receipt of a return, the retailer is obliged to pay return credit, r, back to the customer. It is trivial to extend our model to incorporate nonzero unit return processing cost by simply increasing the value of r accordingly.

The retailer begins the initial period with zero inventory. Then the total cost of ordering in any period has two parts: unit procurement cost, c, and fixed ordering cost, K. Thus, the total cost of ordering O_t units is denoted by $\bar{c}$ and can be stated as follows:

$$\bar{c}(O_t) = \begin{cases} K + cO_t & \text{if } O_t > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Any excess inventory at the end of a period incurs unit holding cost, h. Similarly, any shortage at the end of a period incurs unit backorder cost, b.

We formulate the retailer's multi-period inventory control problem using dynamic programming. The state variable at each period t has two components: the net inventory at the beginning of this period (I_t) and the sales quantity at the previous period (SL_{t-1}). Our objective is to find the optimal net inventory just after replenishment in each period, $S_t := I_t + O_t$, to maximize the expected profit. Using the principle of optimality, one could write the backward recursive formulation for the expected total profit-to-go function $P_t(I_t, SL_{t-1})$ as follows:

$$\begin{aligned} P_t(I_t, SL_{t-1}) = \max_{S_t \geqslant I_t} \{ & p\mathbb{E}_{D_t,R_t}[SL_t] - \bar{c}(S_t - I_t) - r\mathbb{E}[R_t] - J(S_t, SL_{t-1}) \\ & + \mathbb{E}_{D_t,R_t}[P_{t+1}(I_{t+1}, SL_t)] \} \end{aligned} \tag{2}$$

where the first term is the expected revenue from sales, next two terms are the ordering and return costs and the fourth term is the current period's total expected backorder and holding costs which could be written as:

$$J(S_t, SL_{t-1}) = h\mathbb{E}_{D_t,R_t}[(S_t + R_t - D_t)^+] + b\mathbb{E}_{D_t,R_t}[(D_t - S_t - R_t)^+]. \tag{3}$$

In this paper, we use the notations $x^+ := \max\{0, x\}$ and $x^- := \max\{0, -x\}$. During the selling season, the net inventory cannot be salvaged or destroyed. Hence the only constraint on the admissible inventory policies is that $S_t \geq I_t$: the net inventory after replenishment cannot be less than the beginning net inventory. Both parts of the state variable at period $t+1$, I_{t+1} and SL_t, depend on demand and return of period t. Hence the expectation before next period's profit-to-go function in equation (2) is over D_t and R_t. Finally, we assume that there is no discounting as the selling season is comparatively short for many retail products.

The transition of the state variable can be explained in two parts. Given the sequence of events, the net inventory at the beginning of period $t+1$, I_{t+1}, can be simply written as $I_{t+1} = I_t + O_t + R_t - D_t = S_t + R_t - D_t$. Next, the sales quantity at time t can be written as:

$$SL_t = \min\{I_t^+ + O_t + R_t, D_t + B_t\} = \min\{I_t^+ + S_t - I_t + R_t, D_t + I_t^-\} \\ = \min\{S_t + R_t, D_t\} + I_t^-, \tag{4}$$

where the first equality follows from the definitions of O_t and B_t and the second equality follows from the fact that $x = x^+ - x^-$ for any $x \in \mathbb{R}$. Equation (4) formulates that the sales quantity of period t is the minimum of the inventory on-hand plus the replenishment order plus the returns received and current period's demand plus the backordered demand quantity from the previous period. The second equality follows as the backordered quantity from the previous period is the negative part of the net inventory at period t.

At the end of the selling season, which is denoted by period $T+1$, any excess inventory is salvaged at a unit profit of s and any shortage is purchased and delivered to the customers. Following Porteus (1971), we assume that no fixed cost is charged for this last order. We account for any returns of the satisfied backorder demand after the end of the season by including the return cost and the salvage profit due to these returns. Thus the terminal expected profit could be written as follows:

$$P_{T+1}(I_{T+1}, SL_T) = p(I_{T+1})^- + s\mathbb{E}_{R_{T+1}}[(I_{T+1} + R_{T+1})^+] - c\mathbb{E}_{R_{T+1}}[(I_{T+1} + R_{T+1})^-] \\ -r\mathbb{E}[R_{T+1}] + \alpha(s-r)(I_{T+1})^- \tag{5}$$

4 Structure of the Optimal Inventory Policy

In this section, we present some analytical results based on the formulated problem. First, we provide an alternative cost-centric formulation and show that these two formulations are equivalent. Next, the structure of the optimal inventory policy is analyzed utilizing the alternative formulation.

We choose a profit-centric approach in this paper. Alternatively, one could follow a more traditional cost-centric formulation and study the optimal inventory policy of the retailer that minimizes total expected cost. Specifically, one could use a dynamic programming formulation using cost-to-go and terminal cost functions as stated below:

$$C_t(I_t, SL_{t-1}) = \min_{S_t \geqslant I_t}\{\bar{c}(S_t - I_t) + r\mathbb{E}[R_t] + J(S_t, SL_{t-1}) + \mathbb{E}_{D_t,R_t}[C_{t+1}(I_{t+1}, SL_t)]\}, \tag{6}$$

$$C_{T+1}(I_{T+1}, SL_T) = c\mathbb{E}_{R_{T+1}}[(I_{T+1} + R_{T+1})^-] - s\mathbb{E}_{R_{T+1}}[(I_{T+1} + R_{T+1})^+] + r\mathbb{E}[R_{T+1}] \\ +\alpha(r-s)(I_{T+1})^- \tag{7}$$

Next, we show that these two formulations are basically equivalent.

Proposition 1 *Given any state variable pairs I_t and SL_{t-1} the profit-to-go and cost-to-go functions of the two formulations stated in equations (6,7) and (2,5) have the following relationship:*

$$P_t(I_t, SL_{t-1}) = p(\mu(T-t+1) + I_t^-) - C_t(I_t, SL_{t-1}), \quad \forall 1 \leq t \leq T. \tag{8}$$

All the proofs are relegated to appendix. Proposition 1 implies that for each period t, the profit-to-go function is equal to total revenue from the expected demand during the remainder of the selling season and any backordered demand from previous periods less the cost-to-go function for all values of the net inventory and previous period sales quantity. Hence the optimal ordering decisions are exactly the same under both formulations for any state variable pairs I_t and SL_{t-1}. In particular, one can easily deduce that $P_1(0,0) = p\mu T - C_1(0,0)$. This implies that, due to our assumption of backordered unmet demand, one could easily find the optimal expected profit by subtracting the optimal expected total cost from the revenue of expected total demand during the selling season.

The next proposition provides structural results on the optimal inventory policy for the last period:

Proposition 2 *Let $K = 0$ and $c \geqslant s$. Then the following statements hold:*

i) *$C_T(I_T, SL_{T-1})$ is submodular on $(I_T, -SL_{T-1})$.*
ii) *Let $S_T^*(I_T, SL_{T-1})$ be the set of optimal net inventory after replenishment decisions. Then $S_T^*(.)$ is nondecreasing in I_T and nonincreasing in SL_{T-1}.*

Using Proposition 1 and Proposition 2, it is trivial to show that the following corollary is true:

Corollary 1 *Let $K = 0$ and $c \geqslant s$. Then the following statements hold:*

i) *$P_T(I_T, SL_{T-1})$ is supermodular on $(I_T, -SL_{T-1})$.*
ii) *Let $S_T^*(I_T, SL_{T-1})$ be the set of optimal net inventory after replenishment decisions in the profit-centric formulation. Then $S_T^*(.)$ is nondecreasing in I_T and nonincreasing in SL_{T-1}.*

Corollary 1 shows that the optimal net inventory after replenishment is nondecreasing in the net beginning inventory and nonincreasing in the sales quantity of the previous period. The former result is expected given the zero fixed ordering cost assumption and indestructibility of net inventory. The latter result also holds since a higher level of sales quantity in the previous period implies a higher number of possible returns in the current period. This structural result is quite substantial: One could utilize this result to find the optimal inventory decisions more quickly by narrowing down the search space using the optimal decisions for adjacent values of the state variables.

The structural results hold under two assumptions. Nonzero fixed ordering cost could lead the cost-to-go function to be non-submodular. The assumption of $c \geqslant s$ is satisfied for cases worth studying: Otherwise it is optimal to procure a very large quantity in the last period and salvage them at the end of the selling season for profit. Unfortunately, we could not extend the results to other periods in the selling season, which requires convexity of the cost-to-go function which may not hold for many cases. Hence we resort to numerical experiments to analyse the structure of the optimal policy for interim periods under nonzero fixed ordering cost.

5 Computational Results

In this section, we present the results of an extensive numerical study to better understand and quantify the results of our proposed model. First, we study the structure of the optimal inventory control policy using the model proposed in Section 3. Then, we discuss the value of incorporating detailed return flow into the retailer's inventory control problem and how the problem parameters affect this value.

We refer to the retailer that uses our proposed model in managing her inventory as the *"return-smart"* retailer who keeps detailed records of sales and returns and considers item returns as dependent on the *previous period sales*. As a benchmark, we also consider an alternative retailer who does not track previous period sales explicitly, *ignores the dependency of the return flow on sales* and instead simply uses a net demand approach (i.e., considers only the product demand less the returns). For this *"return-naive"* retailer, it is known that the optimal inventory control policy is a traditional (s,S) policy where an order is given to increase net inventory to S only if the net inventory is less than or equal to s (see, for example, Porteus 1971).

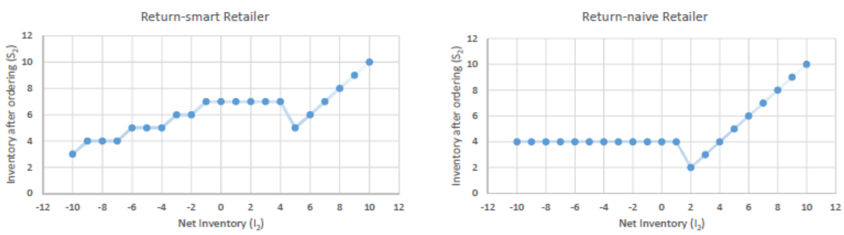
In this computational study, we assume that the demand for each period follows Poisson distribution with mean λ_D. The return-smart retailer considers that each sold item could be returned in the next period with probability α. On the other hand, the return-naive retailer simply uses the net demand in managing the inventory. In our case, one can show that the net demand follows Poisson distribution with mean $(1-\alpha)\lambda_D$.

For the rest of this paper, we consider a particular parameter set, called the base case scenario, to study the differences between the inventory control policies and profits of return-smart and return-naive retailers. Most of the parameters are set following Mitra (2009) which also considers an inventory system with product returns. The mean of the Poisson distribution is taken as $\lambda_D = 4$ and $\alpha = 0.50$, hence the return-naive retailer uses the net demand with mean $(1-\alpha)\lambda_D = 2$. Since the support of the Poisson distribution is unbounded, we truncate the demand from a level such that the cumulative probability exceeds 0.9999. We assume that each period lasts a month and that there are 4 periods in a season $(T = 4)$. Inventory holding cost rate is taken as 5% per unit per month, $b = \$20$ per unit per month, $K = \$25$ per order, $c = \$40$ per unit, and $p = \$80$ per unit. We also assume that there is no salvage value at the end of the period ($s = 0$) and the customer is given full credit for returns and any related processing cost is ignored ($r = p$). In section 5.3, we provide an analysis on the sensitivity of our results on this base case scenario.

5.1 Analysis of the Optimal Inventory Control Policy

We calculate the optimal inventory decisions for the return-smart retailer using our proposed model and also for the return-naive retailer following Porteus (1971). For the base case scenario, the optimal inventory decisions in period 2 for both types of retailers are presented in Figure 1 given that the previous period sales are equal to zero. Let's first compare the form of the optimal policies. As expected, the return-naive retailer follows an (s,S) policy where she places an order only if the net inventory is below or equal to 1 (i.e., re-order point $s = 1$) and increases the inventory to 4 (i.e., order-up-to level $S = 4$). On the other hand, the return-smart retailer does not follow an (s,S) type inventory policy. She places an order only if the inventory level is below or equal to 4, however the order-up-to level is not constant for all values of the net inventory. When the net inventory is below zero (i.e., there is positive backorder from previous periods), the order-up-to level decreases as the number of backorders increases. This could be seen as counter-intuitive at first sight as the retailer could be expected to order more especially when the backordered demand is high. However, the return-smart retailer knows that each fulfilled backordered demand is going to turn into sales in the current period which then could be returned with probability α in the next period. Thus she reduces the order size taking into account the higher number of possible returns from these fulfilled backordered demand. Using the same logic, the return-smart retailer keeps the order-up-to level constant when there is no backordered demand as each product in the inventory has to be first sold and then returned, a lower probability event, in order to be used in fulfilling demand in the future periods.

Fig. 1: The optimal inventory decisions of return-smart and return-naive retailers for the base case scenario in period 2 $(SL_1 = 0)$



Comparing the inventory levels below which an inventory order is given, we observe that the return-smart retailer places an order whenever the net inventory is at or below 4 which is much smaller for the corresponding threshold with the return-naive retailer (i.e., 1). This is because the return-smart retailer expects a higher demand rate of 4 assuming no returns will arrive in period 2 as $SL_1 = 0$. However, the return-naive retailer ignores the sales-dependency of the returns and expects a lower net demand rate of $0.5 * 4 = 2$. This phenomenon also results in the return-naive retailer ordering less (4) than the return-smart retailer (7) when she orders and there is no backorder. Thus we expect the return-naive retailer to have higher backorder costs and higher fixed order cost due to increased order frequency to compensate for the increased backorders. We also find similar observations for the order level in the first period: The return-smart retailer orders 8 units whereas the return-naive retailer orders only 6 units. Later on, whenever the inventory decreases below the re-order point, the return-naive retailer always orders less compared to the return-smart retailer.

Fig. 2: The optimal inventory levels after ordering of the return-smart retailer for the base case scenario in period 2

Previous Period Sales (SL_1)

Net Inventory (I_2)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-10	3	3	3	2	2	1	1	0	0	-1	-1	-2	-3	-3	-4	-4	-5	-5	-6	-6	-7
-9	4	3	3	2	2	1	1	0	0	-1	-1	-2	-2	-3	-3	-4	-4	-5	-5	-6	-6
-8	4	4	3	3	2	2	1	1	0	0	-1	-1	-2	-2	-3	-3	-4	-4	-5	-5	
-7	4	4	3	3	2	2	1	1	0	0	-1	-1	-2	-2	-3	-3	-4	-4			
-6	5	4	4	3	3	2	2	1	1	0	0	-1	-1	-2	-2	-3	-3				
-5	5	5	4	4	3	3	2	2	1	1	0	-1	-1	-2	-2	-3					
-4	5	5	4	4	3	3	2	2	1	1	0	0	-1	-1							
-3	6	5	5	4	4	3	3	2	2	1	1	0	0								
-2	6	6	5	5	4	4	3	3	2	1	1										
-1	7	6	6	5	4	4	3	3	2	2											
0	7	6	6	5	5	4	4	3													
1	7	6	6	5	5	4															
2	7	6	6	5	5																
3	7	6	6																		
4	7																				
5																					
6																					
7																					
8																					
9																					
10																					

NO ORDER IS PLACED IN THIS AREA

Given the effect of net inventory level on inventory decisions, next we investigate the effect of previous period sales. By definition, the return-naive retailer does not use this information. For the return-smart retailer, Figure 2 shows the optimal inventory levels after ordering in period 2 as a function of the net inventory level before ordering and the previous period sales. In the grey shaded area (bottom right) no replenishment order is given. In the white area, the return-smart retailer places a positive replenishment order. For a fixed net inventory level, the order quantity decreases monotonically in the previous period sales. The amount of returns in the current period increases stochastically in the previous sales quantity; thus the retailer orders less expecting a larger number of returns.

5.2 Value of Using Sales-Dependent Returns

In this section, we quantify the value of considering sales-dependent returns in making inventory decisions. Specifically, we calculate the improvement in the profit of the return-smart retailer versus the return-naive one. Given the optimal inventory decisions of both retailers as discussed in the previous subsection, we evaluate the expected profit of both retailers using the same Monte-Carlo simulation for both. The Monte-Carlo simulation generates a random demand value for each period in the horizon and using each retailer's inventory policy, evaluates the resulting average total profit across simulation runs. Clearly, the expected profit of the return-smart retailer is asymptomatically the same if one uses simulation instead of the dynamic programming model of Section 3. Our dynamic programming formulation could be used for the return-naive retailer as well, however we specifically employ simulation to be able to calculate the components of the total profit and better understand the underlying factors for differences in the profits. To minimize any variation due to the generated random numbers, we use identical random demand values while running simulations. Finally, we compare the evaluated total profit of the return-smart retailer for the whole season to that of the return-naive one, assuming zero initial inventory. In this subsection, we use the base case scenario parameters and conduct a one-way sensitivity analysis in the next subsection.

In our Monte Carlo simulation, we determine to use $50,000$ samples. After 10 replications, we find that the standard deviation is $\$0.31 (CV = 0.0031)$, supporting our choice of the number of samples. Due to our assumption that all backordered demand is fulfilled, the total revenues of both retailers are exactly the same. On the other hand, even though both retailers face the same demand realization, they order according to their separate inventory control policies. Thus they may have different sales and return quantities in each period and eventually different total profits from each other.

For the base case scenario, Table 3 provides the total expected profits of both retailers as well as their cost breakdowns. It has four sections: The first section shows a breakdown of the total cost during the selling season into procurement, ordering, holding, and backorder costs. The next section presents the costs incurred after the selling season ends that include the final purchasing cost of remaining backorders, the return credits for these fulfilled backorders, the salvage value of on-hand inventory, if any, after the remaining backorders are fulfilled and the sales revenue of the fulfilled backorders. The third section displays the net revenue after the returns are credited from the total revenue during the selling season. Finally, in the last section, we deduct both total season and end-of-season costs from the net revenue to find the total profit of each retailer.

Table 3 shows that the return-smart retailer has a significant 23% increase in profit compared to the return-naive one. The latter retailer has higher costs in all categories except the holding cost. The underlying reason for this observation stems

Table 3: Total profit breakdowns for both retailer types using base case scenario

Retailers	Procurement Cost B1	Ordering Cost B2	Holding Cost B3	Backorder Cost B4	Total Season Cost B=B1+B2+B3+B4	End-of-Season Cost C	Season Revenue A1	Return Credit A2	Season Net Revenue A=A1-A2	Total Profit A-B-C
Return-naive	399.32	50.41	12.78	50.85	513.37	113.39	1,182.27	453.03	729.24	102.49
Return-smart	380.69	36.59	17.11	47.23	481.62	98.14	1,163.31	457.09	706.21	126.45

from the optimal inventory decisions studied in the previous subsection: The return-naive retailer never considers the incoming returns and orders more frequently in smaller quantities. The total order quantity during the season is also slightly (5%) higher. However, the timing of these orders is not perfect: We find that she also has a worse cycle service level (73% vs 76%) and a product fill rate (89% vs 90%) compared to the return-smart retailer. The return-naive retailer faces more backorders leading to lower service levels as expected. The return-smart retailer orders fewer units at the right time since she considers the possible returns and treats them as a secondary supply source. Moreover, the return-naive retailer has to satisfy a higher number of backorders at the end of the season and then has to pay for any return credits associated with these sales without being able to resell.

On a final note, we observe that using the dynamic programming formulation of Porteus (1971), the return-naive retailer evaluates her optimal profit as \$219.38, which is 114.05% higher than the actual profit (\$102.49) she would get under sales-dependent returns. This level of overestimation would be problematic for many reasons including budget planning. Thus another advantage of our model is the correct evaluation of the actual profit resulting from a chosen inventory control strategy.

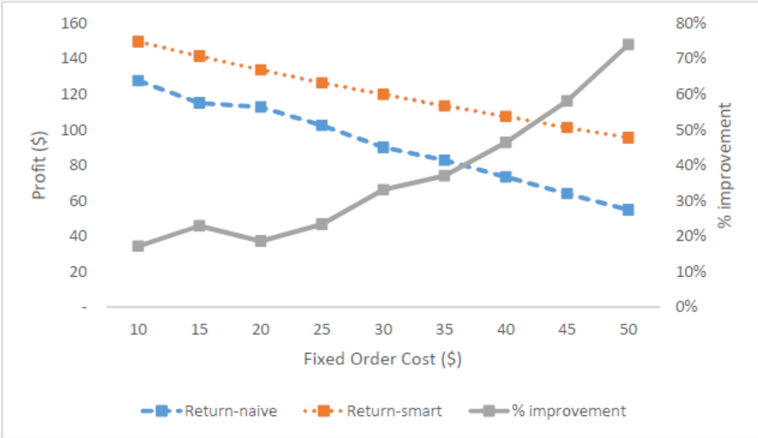
5.3 Sensitivity Analysis

For the base case scenario, we find that the value of incorporating sales-dependent returns in finding the optimal inventory policy is 23%. In this section, we present a one-way sensitivity analysis on this value with respect to problem parameters. For each parameter, we modify the number used in the base case scenario and report the change in the percentage difference in the retailers' profits.

Figure 3 provides the results of sensitivity analysis with respect to the fixed order cost. As expected both retailers' profits decrease as the fixed cost increases. However, the decrease in the return-naive retailer's profit is steeper as this retailer orders more frequently as compared to the return-smart one. The value of incorporating sales-dependent returns increases to more than 70% when the fixed cost increases to 50\$.

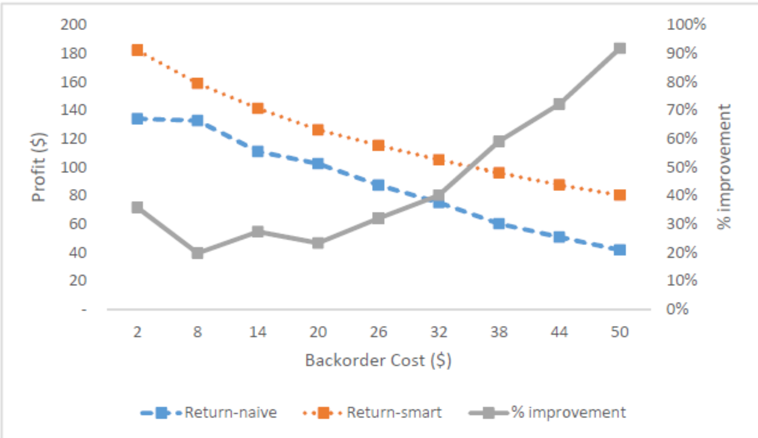
The effect of the backorder cost on the value of incorporating sales-dependent returns is shown in Figure 4. There are two main observations: (1) When the backorder cost is as low as the holding cost (i.e., $h = b = \$2$), the return-naive retailer places no order in the first period to save from the fixed order cost. As a result, she does not sell any products in the first period and loses the opportunity of incoming returns in the second period. On the other hand, the return-smart retailer places a positive initial order and uses the returns in the following periods as a second source of supply

Fig. 3: Value of incorporating sales-dependent returns as the fixed order cost varies



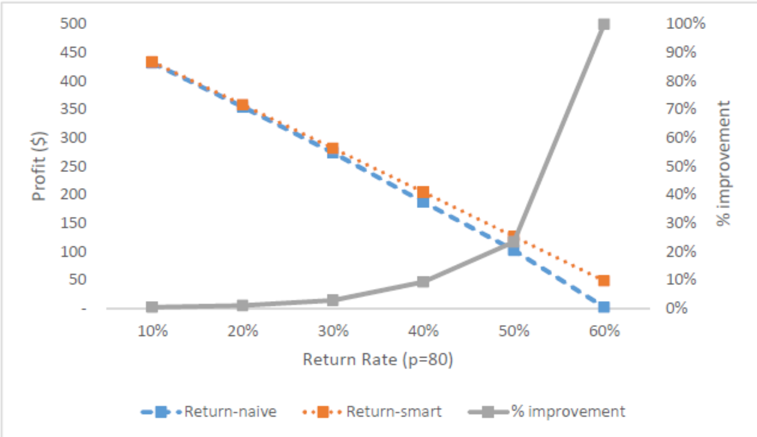
resulting in a 36% profit gap between the two retailers. (2) When the backorder cost is quite large, the return-naive retailer orders more frequently than necessary to save from backorder cost. Thus she incurs high fixed order cost and the percentage profit improvement of incorporating sales-dependent returns could be as high as 70%. For moderate values of the backorder cost, the percentage profit difference is lower.

Fig. 4: Value of incorporating sales-dependent returns as the backorder cost varies



The significant effect of the return probability is evident from Figure 5 which shows the percentage change in the retailer's profit as the return probability (α) varies. The value of incorporating sales-dependent returns increases monotonically with α: With higher values of the return probability, the return-naive retailer chooses a lower initial order quantity expecting a lower net demand. However, the actual demand is much higher in the first period which leads to high backorder costs, low sales, and low returns in each period which ends with a loss at the end of the season.

Fig. 5: Value of incorporating sales-dependent returns as the return probability varies



We conduct additional one-way sensitivity analysis for the remaining cost parameters and the results are presented in Table 4. As the procurement cost increases, the percentage improvement in the retailer's profit reaches 52%. As the profit margin decreases with the higher procurement cost, the return-naive retailer has difficulty in controlling the comparatively higher fixed order costs. Since the total holding cost is the lowest cost category (see Table 3), the percentage improvement in the retailer's profit varies only slightly as the holding cost rate increases from 0.5% to 4%. Finally, we find that the percentage improvement decreases with the salvage value. However, the return-smart retailer's profit is still 4% higher than the return-naive retailer even when the salvage value is the same as the procurement cost.

Table 4: The percentage improvement in the retailer's profit as procurement cost, holding cost and salvage value parameters vary

Parameter	Instance Set	Min	Med	Max
Procurement unit cost	[10,15,...,45]	3%	10%	52%
Holding cost	[1%,2%,...,6%]	23%	27%	29%
Salvage value	[0,10,...,40]	4%	15%	23%

In this paper, we consider a profit-centric approach while determining the ordering decisions. Yet, the resulting service levels could be a separate concern for the retailer. Table 5 shows the percentage difference in the cycle service level and the product fill rate as the problem parameters vary. The percentage differences in both service level measures are most sensitive to the backorder cost as expected from our earlier results. Moreover, the return-naive retailer has better service levels (i.e., the percentage difference is negative) when $\alpha \leq 0.2$ at the expense of higher costs, but both the service level and the profit is smaller for larger values of the return probability.

Table 5: The percentage difference in the service levels as the problem parameters vary

		Type I			Type II		
Parameter	Instance	Min	Med	Max	Min	Med	Max
Procurement unit cost	[10,15,...,45]	3%	6%	7%	1%	1%	2%
Holding cost	[1%,2%,...,6%]	5%	6%	11%	1%	1%	3%
Backorder cost	[8,14,...,50]	3%	6%	19%	0%	1%	6%
Fixed order cost	[10,15,...,50]	2%	8%	9%	0%	2%	2%
Return probability	[10%,20%,...,60%]	-1%	3%	13%	-1%	1%	3%
Salvage value	[0,10,...,40]	4%	6%	7%	1%	1%	2%

6 Extension: Lost Sales Case

Our main formulation introduced in Section 3 assumes that unmet demand is backordered in line with most of the related literature in this field. For online sales, this assumption is even more plausible (see, for example, Mahar and Wright 2009, Bretthauer et al. 2010, and Mahar et al. 2012). However, an alternative assumption would be to assume that any unmet demand is lost (see references in Table 1). Hence in this section, we revisit our research question under the lost sales assumption.

Under the lost sales assumption, the transition of the state variable comprising of the net inventory at the start of period $t+1$ and the previous sales quantity at period $t+1$ could be written as follows:

$$I_{t+1} = (S_t + R_t - D_t)^+ \tag{9}$$

$$SL_t = \min\{S_t + R_t, D_t\} \tag{10}$$

The profit-to-go functions of the dynamic programming formulation also require some modifications. First, expected unmet demand should be multiplied by the lost sales cost. For our base case scenario, we assume that the lost sales cost is zero to focus on the effect of losing sales revenue. Moreover, the terminal profit function is also modified such that it includes only the salvage value of the ending inventory and the credits issued for the returns of the products sold in the last period.

After an analysis of the optimal ordering decisions of both retailers under the base case scenario, we find that the return-smart retailer now follows an (s, S) inventory policy as well. The (s,S) values of the return-smart retailer depend on the previous sales quantity since she considers the return flow as a second source of supply and decreases its orders as the previous period sales increase. During period 2, she follows a $(3,6)$ inventory policy when $SL_1 = 0$ but switches to using a $(1,4)$ inventory policy when $SL_1 \in \{3,4\}$. Similar to our findings with the main model, we find that the return-naive retailer orders fewer quantities compared to the return-smart retailer. For example, she uses a $(2,5)$ inventory policy in the second period. Comparing with the main model, the return-naive retailer has higher (s,S) values under lost sales setting, pointing towards a possible reason for lower profit.

Under the lost sales assumption, the return-smart retailer's profit is 56% higher using the base case scenario as shown in Table 6. Compared to the 23% profit improvement under the backordered demand setting, we conclude that the value of incorporating sales-dependent returns is higher if unmet demand is lost. Notice that the return-naive retailer has a higher revenue, but the increase in costs more than offsets the increase in the revenue. On average the return-naive retailer orders twice, whereas the return-smart retailer only orders in the initial period. In addition, the return-naive retailer also orders more units (10) than the return-smart retailer (8) on average. Since the salvage cost is zero, excess inventory at the end of the selling season is worthless.

Table 6: Total profit breakdowns for both retailer types using the base case scenario and the lost sales assumption

Retailers	Procurement Cost B1	Ordering Cost B2	Holding Cost B3	Total Season Cost B=B1+B2+B3	End-of-Season Cost C	Season Revenue A1	Return Credit A2	Season Net Revenue A=A1-A2	Total Profit A-B-C
Return-naive	407.12	55.46	16.74	479.32	130.44	1,154.63	446.86	707.77	98.01
Return-smart	321.42	25.29	16.01	362.73	87.18	1,031.92	428.91	603.01	153.11

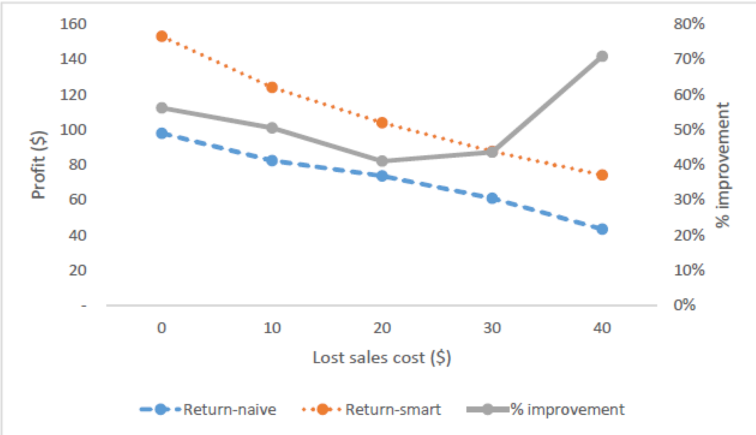
Table 7 shows the results of the one-way sensitivity analysis on the retailer's percentage improvement in profit as the problem parameters vary from the base case scenario. The percentage improvement increases with the procurement unit cost, the holding cost rate, the fixed ordering cost, the return probability and decreases with the salvage value. Compared with the results for the backordered demand setting in Section 4.3, there is a significant increase in the variation of the percentage improvement.

Finally, we investigate the effect of a positive lost sales cost on the percentage profit improvement and present the results in Figure 6. As the lost sales cost increases moderately from zero, the return-smart retailer slightly increases her order frequency and orders in some periods in addition to the initial one. Thus the profit gap between the retailers decreases. When the lost sales cost is very high, however, the return-naive retailer increases order frequency even more to not incur high lost sales cost. The return-smart retailer does not react drastically as she acknowledges possible returns in the subsequent periods. Thus the percentage improvement increases again to more than 70% when the lost sales cost is $40.

Table 7: The percentage improvement in the retailer's profit as the problem parameters vary assuming unmet demand is lost

Parameter	Instance	Min	Med	Max
Procurement unit cost	[10,15,...,45]	4%	14%	128%
Holding cost	[1%,2%,...,6%]	39%	48%	60%
Fixed order cost	[10,15,...,50]	23%	44%	71%
Return probability	[10%,20%,...,60%]	0%	12%	126%
Salvage value	[0,10,...,40]	3%	17%	56%

Fig. 6: Value of incorporating sales-dependent returns as the lost sales cost varies



7 Conclusions

We consider the optimal ordering decisions of a return-smart retailer who faces random returns that depend on previous sales in a finite horizon problem. Using dynamic programming formulation, we search for the best periodic review inventory policy to maximize the expected revenue less the total cost of procurement, ordering, holding, salvage and backorder. Next, we evaluate the optimal expected profit of the return-naive retailer which ignores the dependency of returns on previous sales and simply uses a net demand approach to find her optimal ordering decisions. Comparing the profits of these two retailers quantifies the value of incorporating previous sales on product returns.

Using the base case scenario, we find that the return-smart retailer could obtain a 23% higher profit as compared to the return-naive one. Our sensitivity analysis on the various problem parameters shows that this value could be much higher especially when the fixed ordering cost is high, the backorder cost is very small or high, the return probability is high or the procurement unit cost is high. These documented benefits result as the return-smart retailer uses the returns as a second source of supply and thus limits her order frequency and backorders.

Our results highlight the additional benefit of using return information in planning a retailer's inventory. However, our model requires estimates for the actual demand as well as the return probability which, in turn, requires firms to hold separate meticulous records of both product sales and returns. This requirement could be costly from information technology and labor points of view which is not modeled explicitly in this paper. Moreover, product return process flows should be redesigned to collect this additional information. Our results highlight when and if incurring these additional costs are worthwhile for the retailer.

Customers increasingly opt to return products and with the higher share of e-commerce channel sales, the product return rates are rising above $30-50\%$ in some categories. Due to the Covid-19 pandemic, even more offline sales are substituted by online sales, thus subject to higher return rates. The model presented in this paper is especially important for these retailers who face increasingly number of returned products.

In the retail sector, leftover inventory after the selling season ends mostly don't have a salvage value due to rapid trend changes or seasonal reasons. Hence they could be a costly burden for the firm even after markdown sales and could eventually end up in a landfill. According to Constable (2019), five billion pounds of returned goods end up in the US landfills each year and this landfill waste from returns alone contributes 15 million metric tons of carbon dioxide to the atmosphere. Our model that incorporates sales-dependent returns into inventory management decisions results in less leftover inventory (62% in backorder setting and 87% in the lost sales setting) compared to a model that ignores this dependency. Thus our model could indirectly lower the environmental impact of a retailer by limiting landfill waste.

Our work could be extended along several lines. While we focus on the sales-dependency of returns, another alternative is to model the periods in a finer granularity and allow for product return from multiple previous periods with possibly different return rates. Especially with online sales, customers inform retailers about their intended return by filling out an online form. Thus further research could investigate how this advanced return information could be used to manage returns more effectively. Finally, omnichannel extensions with different and possibly correlated return flows should be studied to expand our understanding of this problem which is here to stay.

Acknowledgements

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Appendix

Proof of Proposition 1: We will complete the proof in two steps. First, we will show that the claim of the proposition holds for $t = T$. Next, we will show that the claim holds for $t-1$, assuming it holds for t. These two steps ensures that the claim holds $\forall 1 \le t \le T$.

Step 1: Using the transition of the state variables I_{T+1} and SL_T, we can rewrite the profit-to-go function for period T multiplied by -1 as follows:

$$\begin{aligned}
-P_T(I_T, SL_{T-1}) = & \min_{S_T \geqslant I_T} \{\bar{c}(S_T - I_T) + r\mathbb{E}[R_T] + J(S_T, SL_{T-1}) - \mathbb{E}_{D_T,R_T}[P_{T+1}(I_{T+1}, SL_T)] \\
& - p\,\mathbb{E}_{D_T,R_T}[SL_T]\} \\
= & \min_{S_T \geqslant I_T} \{\bar{c}(S_T - I_T) + r\,\alpha\, SL_{T-1} + h\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] + bE_{D_T,R_T}[(D_T - S_T - R_T)^+] \\
& - \mathbb{E}_{D_T,R_T}[P_{T+1}(S_T + R_T - D_T, \min\{S_T + R_T, D_T\} + I_T^-)] \\
& - p\,\mathbb{E}_{D_T,R_T}[\min\{S_T + R_T, D_T\} + I_T^-]\}
\end{aligned}$$

Replacing the terminal profit function with equation (5) and using the expectation of the binomial distribution, one can further expand $P_T(I_T, SL_{T-1})$ as:

$$\begin{aligned}
-P_T(I_T, SL_{T-1}) = & \min_{S_T \geqslant I_T} \{\bar{c}(S_T - I_T) + r\,\alpha\, SL_{T-1} + h\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] + bE_{D_T,R_T}[(D_T - S_T - R_T)^+] \\
& + c\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^-] - s\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
& + r\alpha\,\mathbb{E}_{D_T,R_T}[\min\{S_T + R_T, D_T\} + I_T^-] - \alpha(s - r)\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^-] \\
& - p\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^- + \min\{S_T + R_T, D_T\} + I_T^-]\}
\end{aligned} \tag{11}$$

Following similar steps, one could also expand the cost-to-go function for period T as follows:

$$\begin{aligned}
C_T(I_T, SL_{T-1}) = & \min_{S_T \geqslant I_T} \{\bar{c}(S_T - I_T) + r\,\alpha\, SL_{T-1} + h\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] + bE_{D_T,R_T}[(D_T - S_T - R_T)^+] \\
& + c\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^-] - s\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
& + r\alpha\,\mathbb{E}_{D_T,R_T}[\min\{S_T + R_T, D_T\} + I_T^-] + \alpha\,(r - s)(S_T + R_T - D_T)^-\}
\end{aligned} \tag{12}$$

Let's define

$$\begin{aligned}
f_T(S_T, I_T, SL_{T-1}) := & \bar{c}(S_T - I_T) + r\,\alpha\, SL_{T-1} + h\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] + bE_{D_T,R_T}[(D_T - S_T - R_T)^+] \\
& + c\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^-] - s\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
& + r\alpha\,\mathbb{E}_{D_T,R_T}[\min\{S_T + R_T, D_T\} + I_T^-] + \alpha\,(r - s)(S_T + R_T - D_T)^-.
\end{aligned} \tag{13}$$

Notice that

$$(S_T + R_T - D_T)^- + \min\{S_T + R_T, D_T\} = -\min\{S_T + R_T - D_T, 0\} + \min\{S_T + R_T, D_T\} = D_T. \tag{14}$$

Hence equation (11) could be rewritten as:

$$\begin{aligned}
-P_T(I_T, SL_{T-1}) = & \min_{S_T \geqslant I_T} \{f_T(S_T, I_T, SL_{T-1}) - p\,\mathbb{E}_{D_T,R_T}[D_T + I_T^-]\} \\
= & \min_{S_T \geqslant I_T} \{f_T(S_T, I_T, SL_{T-1})\} - p(\mu + I_T^-) = C_T(I_T, SL_{T-1}) - p(\mu + I_T^-),
\end{aligned} \tag{15}$$

where the first equality follows from the fact that the second term inside the minimization function does not depend on the decision variable S_T and the second equality uses the expanded form for the cost-to-go function in equation (12). Thus completing the proof that the claim of the proposition holds for $t = T$.

Step 2: Next we are going to show that the claim holds for period $t-1$ assuming it holds for period t. In other words assume that the following is true for all I_t and SL_{t-1}:

$$P_t(I_t, SL_{t-1}) = p\left(\mu\left(T-t+1\right) + I_t^-\right) - C_t(I_t, SL_{t-1}).$$

Using this assumption, one could write the profit-to-go function for period $t-1$ as:

$$\begin{aligned}-P_{t-1}(I_{t-1}, SL_{t-2}) = \min_{S_{t-1}\geqslant I_{t-1}} \{&\bar{c}(S_{t-1}-I_{t-1}) + r\,\alpha\,SL_{t-2} + h\,\mathbb{E}_{D_{t-1},R_{t-1}}[(S_{t-1}+R_{t-1}-D_{t-1})^+] \\ &+ b\,\mathbb{E}_{D_{t-1},R_{t-1}}[(D_{t-1}-S_{t-1}-R_{t-1})^+] + \mathbb{E}_{D_{t-1},R_{t-1}}[C_t(I_t, SL_{t-1})] \\ &- p\,\mathbb{E}_{D_{t-1},R_{t-1}}[\mu\left(T-t+1\right) + I_t^-] - p\,\mathbb{E}_{D_{t-1},R_{t-1}}[\min\{S_t+R_t, D_t\} + I_t^-]\}\end{aligned} \tag{16}$$

The cost-to-go function for period $t-1$ is:

$$C_{t-1}(I_{t-1}, SL_{t-2}) = \min_{S_{t-1}\geqslant I_{t-1}} \{f_{t-1}(S_{t-1}, I_{t-1}, SL_{t-2})\}, \tag{17}$$

where

$$\begin{aligned}f_{t-1}(S_{t-1}, I_{t-1}, SL_{t-2}) :=&\bar{c}(S_{t-1}-I_{t-1}) + r\,\alpha\,SL_{t-2} + h\,\mathbb{E}_{D_{t-1},R_{t-1}}[(S_{t-1}+R_{t-1}-D_{t-1})^+] \\ &+ b\,\mathbb{E}_{D_{t-1},R_{t-1}}[(D_{t-1}-S_{t-1}-R_{t-1})^+] + \mathbb{E}_{D_{t-1},R_{t-1}}[C_t(I_t, SL_{t-1})].\end{aligned} \tag{18}$$

Now, using the last two equalities and recalling that $I_t = S_t + R_t - D_t$, equation (16) could be rewritten as:

$$\begin{aligned}-P_{t-1}(I_{t-1}, SL_{t-2}) =& \min_{S_{t-1}\geqslant I_{t-1}} \{f_{t-1}(S_{t-1}, I_{t-1}, SL_{t-2}) \\ &- p(\mu(T-t+1) + \mathbb{E}_{D_{t-1},R_{t-1}}[(S_t+R_t-D_t)^- + \min\{S_t+R_t, D_t\} + I_t^-])\} \\ =& \min_{S_{t-1}\geqslant I_{t-1}} \{f_{t-1}(S_{t-1}, I_{t-1}, SL_{t-2}) - p(\mu(T-t+1) + \mathbb{E}_{D_{t-1},R_{t-1}}[D_t + I_t^-])\} \\ =& \min_{S_{t-1}\geqslant I_{t-1}} \{f_{t-1}(S_{t-1}, I_{t-1}, SL_{t-2})\} - p(\mu(T-t+2) + I_t^-) \\ =& C_{t-1}(I_{t-1}, SL_{t-2}) - p(\mu(T-t+2) + I_t^-),\end{aligned}$$

where the third to last equality follows using an equivalent version of equation (14) for period t, and the second to last equality follows as the second part of the minimization function does not depend on the decision variable S_{t-1}, and the last equality is by equation (17). Thus we complete the proof that the claim of the proposition holds for period $t-1$, assuming it holds for period t.

Lemma 1 *Let $\mathscr{C}_t$ denote the set of feasible states and actions:*

$$\mathscr{C}_t := \{(I_t, -SL_{t-1}, S_t) \mid I_t \in \mathbb{Z}, SL_{t-1} \in \mathbb{Z}^+, S_t \geqslant I_t\}.$$

Then $\mathscr{C}_t$ is a lattice.

Proof of Lemma 1: Let x_t^1 , x_t^2 be elements of $\mathscr{C}_t$.
Consider

$$x_t^1 \wedge x_t^2 := (min(I_t^1, I_t^2),\ min(-SL_{t-1}^1, -SL_{t-1}^2),\ min(S_t^1, S_t^2)).$$

Clearly $min(I_t^1, I_t^2) \in \mathbb{Z}$ and $min(-SL_{t-1}^1, -SL_{t-1}^2) \in \mathbb{Z}^+$.
Moreover, we know that $S_t^1 \geqslant I_t^1$ and $S_t^2 \geqslant I_t^2$.
Thus $min(S_t^1, S_t^2) \geqslant min(I_t^1, I_t^2)$, which implies that $x_t^1 \wedge x_t^2 \in \mathscr{C}_t$.
Next, consider

$$x_t^1 \vee x_t^2 := (max(I_t^1, I_t^2),\ max(-SL_{t-1}^1, -SL_{t-1}^2),\ max(S_t^1, S_t^2)).$$

Now, $max(I_t^1, I_t^2) \in \mathbb{Z}$ and $max(-SL_{t-1}^1, -SL_{t-1}^2) \in \mathbb{Z}^+$.
Also, one can conclude that $max(S_t^1, S_t^2) \geqslant max(I_t^1, I_t^2)$, implying $x_t^1 \vee x_t^2 \in \mathscr{C}_t$. Since both $x_t^1 \wedge x_t^2$ and $x_t^1 \vee x_t^2$ are elements of $\mathscr{C}_t$, we can conclude that $\mathscr{C}_t$ is a lattice.

Proof of Proposition 2:
We first rewrite the cost-to-go function using equations (3), (4), (6), and (7):

$$\begin{aligned}
C_T(I_T, SL_{T-1}) = \min_{S_T \geqslant I_T} \{ & \bar{c}(S_T - I_T) + r\alpha SL_{T-1} + h\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] + b\,E_{D_T,R_T}[(D_T - S_T - R_T)^+] \\
& + c\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^-] - s\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
& + r\alpha\,\mathbb{E}_{D_T,R_T}[\min\{S_T + R_T, D_T\} + I_T^-] - \alpha(s-r)\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^-] \} \\
= \min_{S_T \geqslant I_T} \{ & \bar{c}(S_T - I_T) + r\alpha SL_{T-1} + (h+b)\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] + b(\mu - S_T - \alpha SL_{T-1}) \\
& + (c-s)\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] - c\,\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T - D_T + R_{T+1})] \\
& + r\alpha\,\mathbb{E}_{D_T,R_T}[\min\{S_T + R_T, D_T\} + I_T^-] - \alpha(s-r)\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^-] \}
\end{aligned}$$

where the last equality follows from using the equation $(-a)^+ = a^- = a^+ - a$ for any $a \in \mathbb{R}$.
Now, we derive a set of useful equalities.

(i)

$$\begin{aligned}
\mathbb{E}[R_{T+1}] = \mathbb{E}_{D_T,R_T}[\mathbb{E}[R_{T+1}|D_T, R_T]] &= \mathbb{E}_{D_T,R_T}[\alpha(\min\{S_T + R_T, D_T\} + I_T^-)] \\
&= \alpha\,\mathbb{E}_{D_T,R_T}[S_T + R_T - (S_T + R_T - D_T)^+ + I_T^-] \\
&= \alpha S_T + \alpha^2 SL_{T-1} - \alpha\,\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] + \alpha I_T^-
\end{aligned}$$

(ii)

$$\mathbb{E}_{D_T,R_T}[\min\{S_T + R_T, D_T\} + I_T^-] = S_T + \alpha SL_{T-1} - \mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] + I_T^-$$

(iii)

$$\mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^-] = \mathbb{E}_{D_T,R_T}[(S_T + R_T - D_T)^+] - S_T - \alpha SL_{T-1} + \mu$$

Using (i)-(iii) and rearranging terms gives us:

$$\begin{aligned}C_T(I_T, SL_{T-1}) = \max_{S_T \geqslant I_T} \{ & \mu(b+c-\alpha(s-r)) + I_T^- \alpha(r-c) \\ & + \bar{c}(S_T - I_T) - S_T(b+(1+\alpha)c - \alpha s) \\ & + \alpha SL_{T-1}(r-b-c(1+\alpha)+\alpha s) \\ & + (h+b+\alpha(c-s))\mathbb{E}_{D_T,R_T}[(S_T+R_T-D_T)^+] \\ & + (c-s)\mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T+R_T+R_{T+1}-D_T)^+]\} \\ := & \max_{S_T \geqslant I_T} \{\tilde{C_T}(I_T, SL_{T-1}, S_T)\}\end{aligned}$$

Now, we would like to show that $\tilde{C_T}(I_T, SL_{T-1}, S_T)$ is submodular on $\mathscr{C}_T$. In order to show that, let $x_T^1 := (I_T^1, -SL_{T-1}^1, S_T^1)$ and $x_T^2 := (I_T^2, -SL_{T-1}^2, S_T^2)$ be elements of $\mathscr{C}_T$. We need to prove that:

$$\tilde{C_T}(x_T^1) + \tilde{C_T}(x_T^2) \geqslant \tilde{C_T}(x_T^1 \wedge x_T^2) + \tilde{C_T}(x_T^1 \vee x_T^2). \tag{19}$$

Define $W_T(x_T) := \mathbb{E}_{D_T,R_T}[(S_T+R_T-D_T)^+]$ and $Y_T(x_T) := \mathbb{E}_{D_T,R_T,R_{T+1}}[(S_T + R_T+R_{T+1}-D_T)^+]$ where $x_T \in \mathscr{C}_T$ and $R_T \sim Bin(SL_{T-1}, \alpha)$. Using the definition of $\tilde{C_T}(.)$ and the fact that $y+z = \min(y,z)+\max(y,z)\ \forall y,z \in \mathbb{R}$, rearranging the terms of inequality (19) gives us:

$$\begin{aligned}&\bar{c}(S_T^1 - I_T^1) + \bar{c}(S_T^2 - I_T^2) + (h+b+\alpha c - \alpha s)(W_T(x_T^1) + W_T(x_T^2)) \\ &+ (c-s)(Y_T(x_T^1)+Y_T(x_T^2)) \geqslant \bar{c}(\min(S_T^1,S_T^2) - \min(I_T^1,I_T^2)) \\ &\bar{c}(\max(S_T^1,S_T^2) - \max(I_T^1,I_T^2)) + (h+b+\alpha(c-s))(W_T(x_T^1\wedge x_T^2) + W_T(x_T^1 \vee x_T^2)) \\ &+ (c-s)(Y_T(x_T^1 \wedge x_T^2) + Y_T(x_T^1 \vee x_T^2)).\end{aligned} \tag{20}$$

Part 1: Without loss of generality, assume that $S_T^1 \leqslant S_T^2$. If $-SL_{T-1}^1 \leqslant -SL_{T-1}^2$, then $W_T(x_T^1 \wedge x_T^2) = W_T(x_T^1)$ and $W_T(x_T^1 \vee x_T^2) = W_T(x_T^2)$. Thus $W_T(x_T^1) + W_T(x_T^2) \geqslant W_T(x_T^1 \wedge x_T^2) + W_T(x_T^1 \vee x_T^2)$.

Otherwise $W_T(x_T^1 \wedge x_T^2) = \mathbb{E}_{D_T,R_T^1}[(S_T^1 + R_T^2 - D_T)^+]$ and $W_T(x_T^1 \vee x_T^2) = \mathbb{E}_{D_T,R_T^2}[(S_T^2 + R_T^1 - D_T)^+]$ where $R_T^i \sim Bin(SL_{T-1}^i, \alpha)$. Using Klenke and Mattner (2010), since $SL_{T-1}^1 \leqslant SL_{T-1}^2$, we conclude that R_T^2 dominates R_T^1 under first-order stochastic dominance (FSD).

Consider $u_T(x) := \mathbb{E}_{D_T}[(S_T^2+x-D_T)^+ - (S_T^1+x-D_T)^+]$. Since $S_T^1 \leqslant S_T^2$, $u_T(x)$ is a non-decreasing function. Thus, by Milne and Neave (1994), we can deduce that

$$\mathbb{E}_{D_T,R_T^2}[(S_T^2+R_T^2-D_T)^1 - (S_T^1+R_T^2-D_T)^+] \geqslant \mathbb{E}_{D_T,R_T^1}[(S_T^2+R_T^1-D_T)^1 - (S_T^1+R_T^1-D_T)^+]$$

which implies that $W_T(x_T^2) + W_T(x_T^1) \geqslant W_T(x_T^1 \wedge x_T^2) + W_T(x_T^1 \vee x_T^2)$.

Part 2: Following similar steps of part 1, we now show that

$$Y_T(x_T^2) + Y_T(x_T^1) \geqslant Y_T(x_T^1 \wedge x_T^2) + Y_T(x_T^1 \vee x_T^2).$$

Again, without loss of generality, assume that $S_T^1 \leqslant S_T^2$.

The case with $-SL_{T-1}^1 \leqslant -SL_{T-1}^2$ is obvious. For $SL_{T-1}^1 \leqslant SL_{T-1}^2$, let $R_{T+1}^i(x) \sim Bin(\{S_T^i + x, D_T\} + I_T^-, \alpha)$ for any $x \in \mathbb{Z}^+$. Define

$$V_T(x) := \mathbb{E}_{D_T, R_{T+1}^2}[(S_T^2 + x + R_{T+1}^2(x) - D_T)^+ | x] - \mathbb{E}_{D_T, R_{T+1}^1}[(S_T^1 + x + R_{T+1}^1(x) - D_T)^+ | x]$$

Since $S_T^1 \leqslant S_T^2$, $R_{T+1}^2(x)$ dominates $R_{T+1}^1(x)$ under FSD for any $x \in \mathbb{Z}^+$. Thus it is trivial to see that $V_T(x)$ is a non-decreasing function. Also recalling that R_T^2 dominates R_T^1 under FSD leads to $\mathbb{E}_{R_T^2}[V_T(R_T^2)] \geqslant \mathbb{E}_{R_T^1}[V_T(R_T^1)]$ which further implies that:

$$\mathbb{E}_{R_T^2}[\mathbb{E}_{D_T, R_{T+1}^2}[(S_T^2 + R_T^2 + R_{T+1}^2(R_T^2) - D_T)^+ | R_T^2]] - \mathbb{E}_{R_T^2}[\mathbb{E}_{D_T, R_{T+1}^1}[(S_T^1 + R_T^2 + R_{T+1}^1(R_T^2) - D_T)^+ | R_T^2]]$$
$$\geqslant \mathbb{E}_{R_T^1}[\mathbb{E}_{D_T, R_{T+1}^2}[(S_T^2 + R_T^1 + R_{T+1}^2(R_T^1) - D_T)^+ | R_T^1]] - \mathbb{E}_{R_T^1}[\mathbb{E}_{D_T, R_{T+1}^1}[(S_T^1 + R_T^1 + R_{T+1}^1(R_T^1) - D_T)^+ | R_T^1]]$$

thus showing that $Y_T(x_T^2) + Y_T(x_T^1) \geqslant Y_T(x_T^1 \wedge x_T^2) + Y_T(x_T^1 \vee x_T^2)$.

Using the results obtained in parts 1 and 2, and recalling the assumptions that $K = 0$ and $c \geqslant s$, we can conclude that inequality (20) holds completing the proof that $\tilde{C_T}(I_T, SL_{T-1}, S_T)$ is submodular on $\mathscr{C}_T$.

Following Topkis (1998), we can thus deduce that the set of optimal net inventory after replenishment decisions, $S_T^*(I_T, SL_{T-1})$, is non-decreasing in I_T and non-increasing in SL_{T-1} by definition of $\mathscr{C}_T$ in Lemma 1. Moreover, preservation of submodularity under minimization (see, Topkis 1978) implies that $C_T(I_T, SL_{T-1})$ is also submodular on $(I_T, -SL_{T-1})$.

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