

CHEMICAL THERMODYNAMICS
AND THERMOCHEMISTRY

Estimation of Viscosity of Alloys Using Gibbs Free Energy
of Mixing and Geometric Model

Ali Dogan^a and Hüseyin Arslan^{a,b,*}

^a Science and Art Faculty, Department of Physics, Kahramanmaraş Sutcuimam University, Avsar Campus, Onikisubat/Kahramanmaraş, 46100 Turkey

^b Kahramanmaraş Sutcuimam University, Institute for Graduate Studies in Science and Technology, Department of Material Science and Engineering, Avsar Campus, Onikisubat/Kahramanmaraş, 46100 Turkey

*e-mail: hseyin_arslan@yahoo.com

Received June 3, 2019; revised December 11, 2019; accepted August 27, 2020

Abstract—In the present work, using mixing Gibbs free energies and Chou's general solution model (GSM), by considering the excess activation energies from the binary subsystems, the viscosities of the simple ternary Au–Ag–Cu, Al–Cu–Si, and Fe–Ni–Co and liquid alloys of binary subsystems have been evaluated via well known Chou model and physical models, such as Kaptay, Kozlov–Romanov–Petrov (KRP), and Schick et al. at temperatures 1373, 1375, and 1873 K. A comparison between the evaluated results and experimental values of the Au–Ag–Cu, Al–Cu–Si, and Fe–Ni–Co ternary alloys was carried out. In this study, the success of the application of the aforementioned geometric and physical models to the viscosity calculations of the alloys discussed and the viscosite data are presented to the literature. In order to determine the applicability success, the mean square deviation analysis was performed. According to the values in this table, Schick et al. and KRP models which are derived from the physical quantities among the models discussed provide best description of the viscosity for the Al–Cu–Si and Au–Ag–Cu alloys, respectively.

Keywords: activation energy of viscous flow, mixing Gibbs free energy, geometric models, Chou's model, multi component alloys

DOI: 10.1134/S003602442103002X

1. INTRODUCTION

However, it is time-consuming and expensive to realize the viscosities of ternary or multicomponent melts. Considering this situation, many attempts [1–7] were carried out to predict the viscosity of multi-component system using only thermodynamic parameters. Starting from the Gibbs energies of the binary systems, lots of efforts have been made to determine the Gibbs free energies of the multi-component alloy systems and a number of quasi-empirical models have been proposed to determine energy interaction parameters of binary systems [8–12].

The viscosity of liquid metals has been investigated both experimentally and theoretically for many decades [13, 14]. In addition, for the alloy components treated in this study, there are about twenty of viscosity data about the pure Al, Cu, and Si liquid melts in literature [15–36].

For the binary alloy systems, in some recent studies related to Al–Cu–Si, viscosities of the binary Al–Cu, Cu–Si, and Al–Si systems have been measured using the oscillating cup method [37–39] or the Al–Cu system using a gas bubble viscometer [40].

The density and viscosity of an Al–Cu–Si liquid were measured using an electrical levitation technique [41–43] and an oscillating cup viscometer [23], respectively, and the viscosity was compared to a number of existing thermodynamic models [3, 7]. Moreover, Gruner and Hoyer [44] determined the viscosities of the liquid Cu–Si alloys in the Cu-rich composition range (up to 40 at % Si) at 1273–1473 K.

For the investigated Au–Ag–Cu ternary system, it is seen that there are also some references for liquid alloys viscosities, including both experimental [29, 45, 46] and calculated data [47–49].

Some experimental data are available for the binary subsystems, such as Fe–Co [50–53], Fe–Ni [53–55], and Ni–Co binary melts [53]. Recently, a study was proposed by Sato et al. [53]. It produces measured values for ternary liquid alloys with high precision using an oscillating cup viscometer over the entire composition range from liquidus temperatures up to 1873 K.

The aim of this investigation is to predict the viscosities of the liquid binary and ternary alloys in the systems Au–Ag–Cu, Al–Cu–Si, and Fe–Ni–Co at the related temperatures. The viscosities of the binary alloys were calculated based on the mixing Gibbs

energy model proposed by Seetharaman et al. [56]. The polynomial equations Redlich–Kister–Muggianu were used to describe the variations of excess activation energies of three binary systems. The geometric model mentioned above were used to predict the excess viscosities of ternary alloys. The calculated results of the ternary alloys except for Fe–Ni–Co were compared with the experimental values by considering the applicability of Chou’s models. Moreover, a comparison was also given between this model and physical models, such as Kaptay, KRP, and Schick et al. models.

2. BACKGROUND RELATED TO THE MODELS

For a liquid mixture, Eyring’s absolute rate theory for calculating the viscosity of has been successfully applied to metallic systems as well as to various ionic systems, and its expression can be given as follows:

$$\eta = \frac{\rho h N_A}{M} e^{\frac{E_a}{RT}}, \quad (1)$$

where η is the dynamic viscosity of the alloy, h is the Plank constant, N_A is the Avogadro constant, ρ is the density of the liquid alloy, M is the molar mass of the alloy, E_a is the activation energy for the flow of a viscous liquid, T is the absolute temperature, and R is the universal gas constant.

In the case of high-order metallic systems, the molar mass and density of the melt may be written as follows:

$$\rho = \sum_{i=1}^N x_i \rho_i \quad (2)$$

and

$$M = \sum_{i=1}^N x_i M_i. \quad (3)$$

It is well-known that the activation energy of a liquid is regarded as a function of both the temperature and the composition of the solution. Its activation energy should firstly be known to determine theoretically the viscosity of a liquid alloy. On the other hand, it is not an easy task to obtain the activation energies for alloys. An expression related to this for the binary liquid alloys in the literature was suggested [6]:

$$E_A = \sum X_i E_{a_i} + G_M + 3RTX_A X_B. \quad (4)$$

Here, E_{a_i} is the activation energy of the i th component and G_M is Gibbs energy of the mixture, X_A and X_B are the molar fractions of the binary alloys. The expression of Gibbs energy of the mixture can be written as a function of the activities associated with the components of the liquid binary alloys:

$$G_M = RT(X_A \ln a_A + X_B \ln a_B). \quad (5)$$

Similar to the viscosity calculation of binary alloy, the activation energy of ternary alloy can be calculated by Eq. (4) considering the excess activation energy $E_a^{\text{Exc}} = E_A - \sum X_i E_{a_i}$ and activation energies of three pure metals.

It is possible to write the excess activation energy for the binary alloys in the form:

$$E_a^{\text{Exc}} = X_A X_B \left[\sum_{i=1}^2 A_{ij}^k (X_A - X_B)^k \right], \quad (6)$$

where A_{ij}^k are the binary interaction coefficients related to the binary A–B subsystem. The excess activation energies of three binary subsystems of the Au–Ag–Cu, Al–Cu–Si, and Fe–Ni–Co ternary system can easily be calculated from Eq. (6) which are fit using polynomial equation described by this expression.

Recently, Chou proposed a general solution model to predict the thermodynamic properties of a ternary system from the investigated three boundary system. The most attractive property of this new model is bypassing the traditional methods and subsuming symmetric and asymmetric models in a more general approach. Here, in order to describe the excess viscosity of ternary alloy, Chou model was also extended to the activation energy prediction of the ternary system from the three boundary systems, which this model has been considered shortly here, as shown in Eq. (7):

$$\eta^E = \sum_{i,j=1, (i \neq j)}^k W_{ij} \eta_{ij}^E, \quad (7)$$

where $W_{ij} = x_i x_j / X_{i(i,j)} X_{j(i,j)}$; $X_{i(i,j)} = X_i + \sum_{k=1, k \neq i, j}^m X_k \xi_{i(i,j)}^k$. Here, $\xi_{i(i,j)}^k$ is defined as similarity coefficients. The scheme of Chou’s model has been described in detail in [12, 49, 57–65].

Furthermore, the graphs in the alloys treated become difficult to distinguish between one another due to the overlap of the graphs plotted for the models. Therefore, the mean square deviation, S , analysis was performed for the data of the ternary alloys systems in which its expression is written as:

$$S = (1/N) \left\{ \sum_{i=1}^N (\eta_{\text{exp},i}^{\text{ex}} - \eta_{\text{cal},i}^{\text{ex}})^2 \right\}^{1/2}. \quad (8)$$

where $\eta_{\text{exp},i}^{\text{ex}}$ and $\eta_{\text{cal},i}^{\text{ex}}$ denote the experimental and calculated values of the viscosity, while N represents the number of the experiments.

3. RESULTS AND DISCUSSION

In order to obtain the accurate values of the activities, the binary excess Gibbs energy interaction parameters of the binary liquid alloys Si–Cu, Si–Al,

Table 1. Data of the viscosity, density, and molar mass used in this study

Metal	Viscosity, mPa s	Ref.	Density, g/cm ³	Ref.	Molar mass, g/mol	Ref. temp., K
Al	0.7157	[75]	2.2407	[76]	26.981	1375
Cu	4.3864	[75]	8.018	[76]	63.546	1375
Si	0.7516	[77]	2.6314	[76]	28.981	1375
Ag	3.19	[29, 78]	9.1853	[29, 78]	107.88	1373
Au	5.13	[29, 78]	7.7165	[29, 78]	197	1373
Fe	5.29	[59, 79]	6.97	[80]	55.85	1873
Ni	3.82	[59, 79]	7.75	[80]	58.69	1873
Co	5.1052	[59, 79]	7.638	[80]	58.94	1873

and Cu–Al have been adapted from those in refs, respectively [66, 67]. The binary excess Gibbs energy interaction parameters of the binary liquid alloys Ag–Au, Au–Cu, and Cu–Ag have been adapted from those in refs, respectively [68–70] while the binary excess Gibbs energy interaction parameters of the binary liquid alloys Fe–Ni, Ni–Co, and Co–Fe have been adapted from that in [71]. The data of the viscosity, density and molar mass are given in Table 1 and then the activation energies for Al, Cu, and Si are obtained with combining Eq. (1), and the values for activation energy are calculated as 35793.6, 51737.6, and 35332.8 (J/mol), for Al, Cu, and Si at 1375 K respectively. Moreover, the activation energy values are calculated as 57688.7, 52529.4, and 51658.9 for Au, Ag, and Cu at 1373 K. In addition, the activation energy values are also calculated as 72615.4, 66666.1, and 71474.3 for Fe, Ni, and Co at 1873 K.

Using the references concerning excess energy mentioned in the previous paragraph, the mixing Gibbs energy curves for binary A–B-subsystems of the

liquid alloys according to the second components plotted in Figs. 1–3.

On the other hand, according to the calculated activation energies of binary alloys and the activation energies of pure metals used in the ternary alloys, the calculated the excess activation energies (Figs. 4–6) are given in Tables 2–5 which are fit using polynomial equation described by Eq. (6) and the calculated values of the Redlich–Kister parameters are listed in Table 2. In order to describe the excess viscosity of ternary alloy from the measured viscosities of binary subsystems, here Chou’s model is used to predict the excess activation energy instead of excess Gibbs free energy concerning the viscosity of alloys of binary subsystems. Chou’s model has been considered to calculate the viscosity of ternary alloys using the experimental excess viscosity values for the alloys of binary subsystems as in [12, 49, 57–65]. To calculate the viscosity values dealing with the physical models, the concentration dependence of the enthalpy of mixing for liquid Al–Cu–Si alloys has been taken from [72]. In addition, the concentration dependence of the

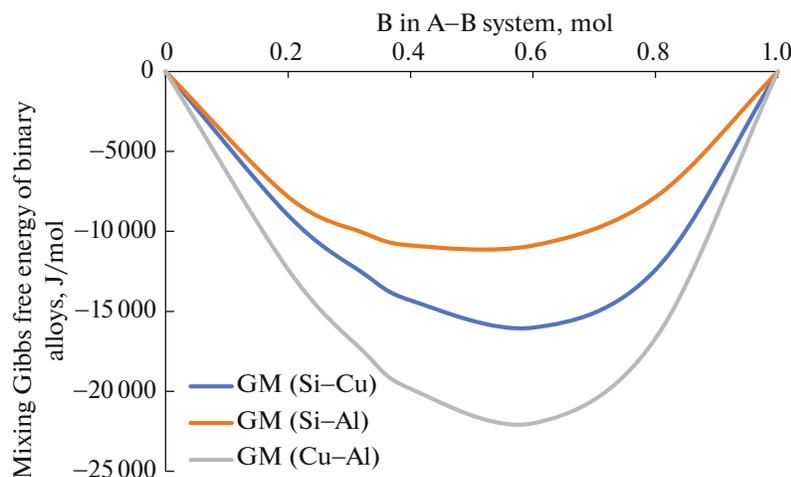


Fig. 1. Mixing Gibbs free energy for binary A–B systems of the liquid alloys Al–Cu–Si with X_B at 1375 K. Here, GM stands for the mixing Gibbs free energy.

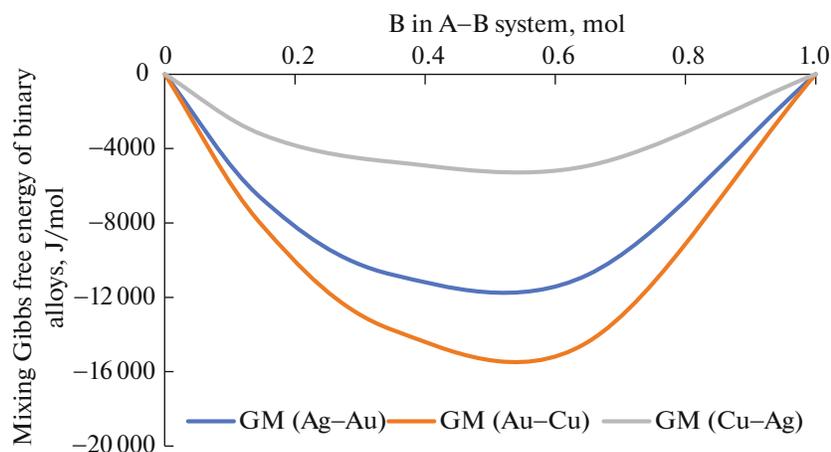


Fig. 2. Mixing Gibbs free energy for binary A–B systems of the liquid alloys Au–Ag–Cu with X_B at 1373 K. Here, GM stands for the mixing Gibbs free energy.

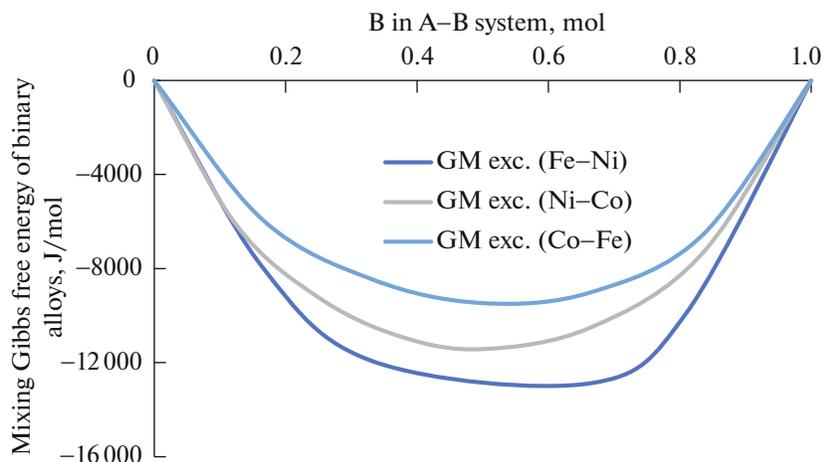


Fig. 3. Mixing Gibbs free energy for binary A–B systems of the liquid alloys Fe–Ni–Co with X_B at 1873 K. Here, GM stands for mixing the Gibbs free energy.

enthalpy of mixing for liquid Au–Ag–Cu alloys has been taken from [73]. Moreover, the concentration dependence of the enthalpy of mixing for liquid Fe–Ni–Co alloys has been taken from [71].

As for the binary liquid alloys Si–Cu, Si–Al, and Cu–Al, from Eqs. (1)–(7), it is easy to obtain the viscosities of binary alloys just mentioned above. As shown in Fig. 7, since the viscosity values measured by Schick et al. and other authors [37, 39] between 873 and 1500 K were quite large with respect to the viscosity values calculated at 1375 K in this study according to the composition Cu ($0 \leq x_{\text{Cu}} \leq 0.2$) in the liquid Al–Cu alloys, the viscosity values associated with the experimental and the other theoretical models in the liquid Al–Cu alloys were not added to this figure. The prediction values of viscosity for Al–Si according to the Si composition slightly change. However, the viscosity results of the model treated in this study are very

small in range of the Si composition between 0 and 0.4 compared to the experimental results [31, 45] on viscosities in the order of 0.8 and 1.2 mPa s for the range 973 and 1073 K. The prediction values of viscosity for liquid Cu–Si alloys according to the Si composition become slightly constant. The viscosity values measured by the other authors [44, 77] between 1273 and 1473 K were quite large with respect to the viscosity values calculated at 1375 K in this study according to the composition Si ($0 \leq x_{\text{Cu}} \leq 0.4$). Therefore the experimental values for the liquid Cu–Si alloys were not added to Fig. 7. The viscosity of ternary Al–Cu–Si liquid alloys along the section $\text{Al}_x(\text{Cu}_{50}\text{Si}_{50})_{1-x}$ at 1375 K is shown in Fig. 8 as the lines with calculated values based on the models.

Figure 9 displays the calculated viscosities of liquid Au–Ag, Au–Cu, and Ag–Cu alloys at 1373 K. For a comparison, the experimental values for these alloys

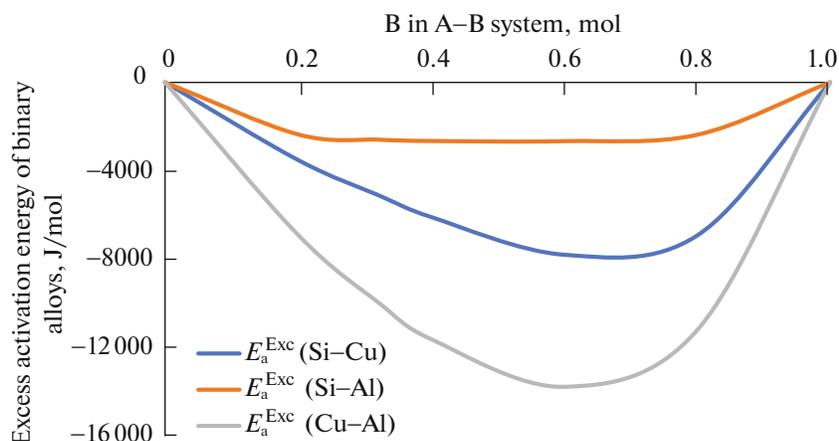


Fig. 4. The excess activation energies for three binary A–B subsystem alloys of the liquid alloys Al–Cu–Si with X_B at 1375 K. Here, E_a^{Exc} stands for the excess activation free energy.

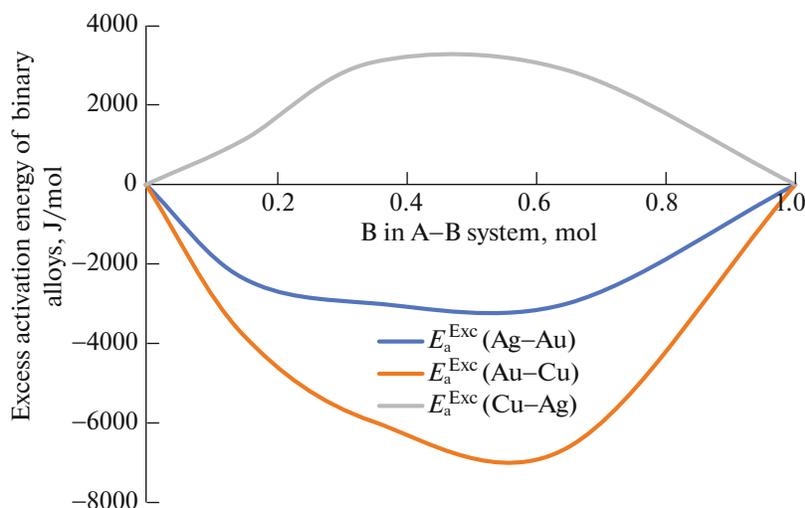


Fig. 5. The excess activation energies for three binary A–B subsystem alloys of the liquid alloys Au–Ag–Cu with X_B at 1373 K. Here, E_a^{Exc} stands for excess activation free energy.

are taken from [45]. By comparing the predicted values by the based on the mixing Gibbs energy model proposed by Seetharaman et al. [6, 56] and experimental data [29, 45] for the alloy Au–Cu, it can be seen that the model mentioned above can not reproduce the experimental data at the composition regions outside of the pure elements, while for the alloys Au–Ag and Ag–Cu, it can also be seen that the model mentioned above can reproduce the experimental data at the entire composition range.

Figure 10 displays the dependence of viscosity of ternary Au–Ag–Cu alloy on composition at 1373 K for section $x_{\text{Ag}}/x_{\text{Cu}} = 0.543$ along with calculated values based on the models. Experimental values for this alloy is taken from [45]. On the other hand, the calculated viscosities for three liquid alloys of constitutive

binary subsystems, Fe–Ni, Fe–Co, and Ni–Co, at 1873 K are given in Fig. 11. For this purpose, the experimental values of these alloys are taken from [53]. The evaluated values by Seetharaman model in this work seems a little lower than the measured results of Sato et al., while shows a good agreement with those of Krieger et al. [74]. It can also be seen for the liquid Fe–Ni, Fe–Co, and Ni–Co alloys at 1873 K that the model concerning Seetharaman model mentioned above can reach a good agreement with the experimental data at the entire composition range (0–1) of the second elements.

Figure 12 displays the dependence of viscosity of ternary Fe–Ni–Co alloy on composition at 1873 K for section $x_{\text{Ni}}/x_{\text{Co}} = 3/1$ along with calculated values based on the models. From viscosity graphs for the ter-

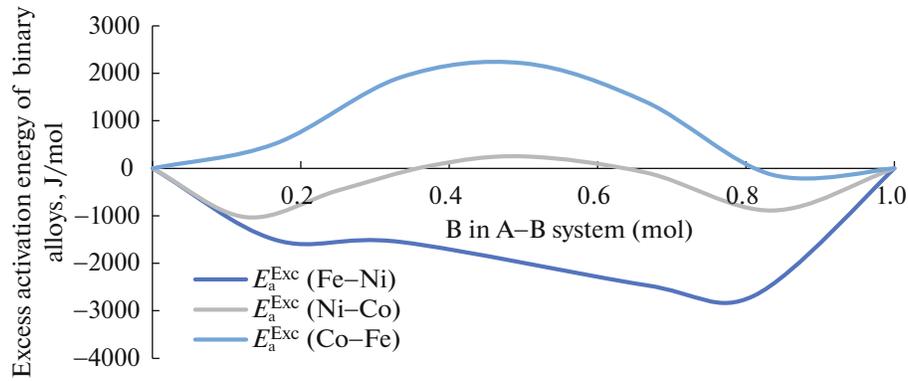


Fig. 6. The excess activation energies for three binary A–B subsystem alloys of the liquid alloys Fe–Ni–Co with X_B at 1873 K. Here, E_a^{Exc} stands for excess activation free energy.

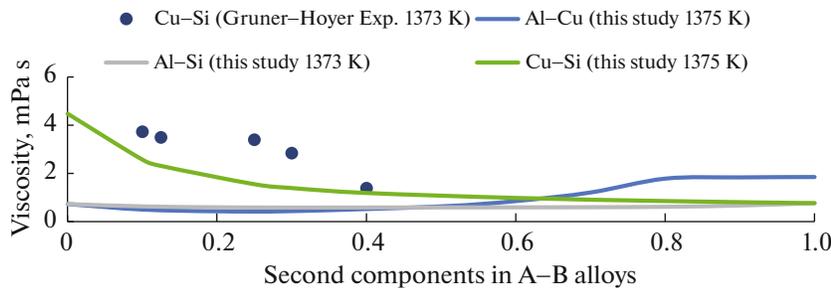


Fig. 7. Calculated viscosities in liquid Al–Cu, Al–Si, and Cu–Si alloys at various temperatures. Experimental values of Cu–Si alloys are taken from [44].

nary alloys, it can be observed that all models are in agreement one another mutually. A comparison of viscosity results for ternary alloys is given in the conclusions.

Table 2. Redlich–Kister parameters determined for activation energies for the A–B binary subsystem alloys of Au–Ag–Cu, Al–Cu–Si, and Fe–Ni–Co at 1373, 1375, and 1873 K in this work, respectively

Alloy system	A_{ij}^0	A_{ij}^1	A_{ij}^2	A_{ij}^3
Al–Cu	–52390.75	–22550.5	–11561.75	
Al–Si	–10544	–169	–11562.2	
Cu–Si	–28413.5	–17850.5	–11562	
Fe–Ni	–7130.625	6266.375	–17267.875	–775.875
Ni–Co	1327.375	648.125	–17181.875	–2121.625
Co–Fe	9093.5	3519	–16813.5	
Au–Ag	–11936	0	–13397.1	
Au–Cu	–26434.5	4750.5	–13397	
Cu–Ag	14268.5	1524.5	–13397.4	

CONCLUSIONS

In this study, instead of using the excess viscosities of the two subbinary alloys the composition, considering the excess activation energies from the subbinary systems, a viscosity model based on the Seetharaman model was compared with those of some physical models and the results of experimental studies associated with Al–Cu–Si and Au–Ag–Cu at 1375 and 1373 K, respectively.

Using the mixing Gibbs energy and the activation energies of pure metals proposed in this study, the viscosities of binary alloys at the related temperatures are obtained and using the calculated excess activation energies, the viscosity values of three simple ternary alloy systems are calculated from Chou’s model along with KRP, Kaptay, and Schick et al. model.

Firstly, before calculating the viscosities of the alloys, the values for activation energy must be calculated. Using Table 1 and Eq. (1), the values for activation energy were calculated as 35793.6, 51737.6, and 35332.8 (J/mol), for Al, Cu, and Si at 1375 K respectively. Moreover, the activation energy values were calculated as 57688.7, 52529.4, and 51658.9 for Au, Ag, and Cu at 1373 K. In addition, the activation energy

Table 3. The calculated activation excess energies and activation energies (J/mol) for the A–B binary subsystem alloys of Al–Cu–Si with X_B at 1375 K

X_2 in A–B	E_a^{Exc} (Si–Cu)	E_a^{Exc} (Si–Al)	E_a^{Exc} (Cu–Al)	E_a (Si–Cu)	E_a (Si–Al)	E_a (Cu–Al)
0	0	0	0	35332.6	35332.6	51737.3
0.2	–3510.6	–2348.9	–6895.7	35103.0	33075.9	41652.9
0.333	–5117.6	–2588.0	–9989.1	35501.0	32893.2	36610.9
0.4	–6085.0	–2645.0	–11613.8	35809.6	32872.0	33746.0
0.6	–7791.0	–2653.6	–13771.1	37384.5	32955.5	28400.0
0.8	–6922.7	–2366.2	–11210.2	41533.8	33335.2	27772.1
1	0	0	0	51737.3	35793.4	35793.4

Table 4. The calculated activation excess energies and activation energies (J/mol) for the A–B binary subsystem alloys of Au–Ag–Cu with X_B at 1373 K

X_2 in A–B	E_a^{Exc} (Ag–Au)	E_a^{Exc} (Au–Cu)	E_a^{Exc} (Cu–Ag)	E_a (Ag–Au)	E_a (Au–Cu)	E_a (Cu–Ag)
0	0	0	0	52529.3	57024.9	51661.2
0.15	–2358.8	–3783.3	1118.2	50844.9	56220.4	52909.7
0.35	–2989.7	–5963.9	3075.6	51113.0	55147.6	55040.7
0.65	–2989.7	–6612.3	2867.5	52461.7	53538.5	55093.1
1	0	0	0	57024.9	51661.2	52529.3

Table 5. The calculated activation excess energies and activation energies (J/mol) for the A–B binary subsystem alloys of Fe–Ni–Co with X_B at 1873 K

X_2 in A–B	E_a^{Exc} (Fe–Ni)	E_a (Fe–Ni)	x^2	E_a^{Exc} (Co–Fe)	E_a (Co–Fe)	x^2	E_a^{Exc} (Ni–Co)	E_a (Ni–Co)
0	0	72615.1	0	0	71474.1	0	0	66665.9
0.165	–1503.8	70129.8	0.165	517.5	72180.0	0.125	–1029.7	66237.3
0.335	–1552.5	69069.8	0.335	1917.4	73774.0	0.25	–466.5	67401.6
0.665	–2461.2	66197.8	0.5	2210.9	74255.8	0.375	57.9	68527.0
0.81	–2687.8	65108.5	0.665	1400.0	73633.1	0.5	253.6	69323.8
1	0	66666.0	0.835	–132.1	72294.8	0.665	–89.1	69774.4
			1	0	72615.1	0.835	–890.6	69790.2
						1	0	71474.3

values were also calculated as 72615.4, 66666.1, and 71474.3 for Fe, Ni, and Co at 1873 K. The viscosity data calculations for physical models such as Chou, Zivkovic–Manasijevec, KRP, Kaptay, and Schick et al. are also performed in this study.

It is seen from the viscosities of Al–Cu–Si liquid alloys for Schick et al. and this study along the sec-

tion $\text{Al}_x(\text{Cu}_{50}\text{Si}_{50})_{1-x}$ at 1375 K seen in Fig. 8 have minimums in the composition range between 0.8–0.9 of the composition Al. These minimums seen in this figure are in agreement with experiment results. Moreover, the viscosities for KRP and Kaptay models decrease slowly with increasing Al concentration.

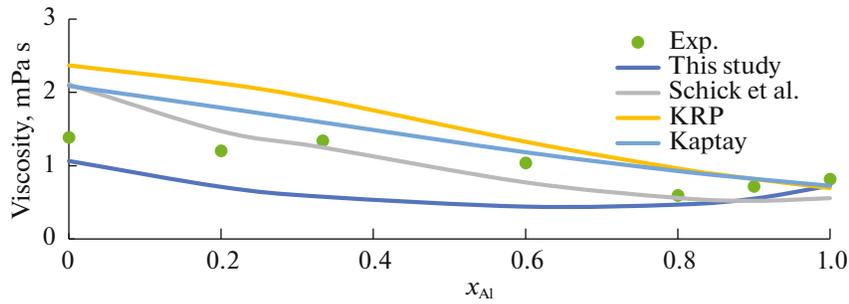


Fig. 8. The viscosity of ternary Al–Cu–Si liquid alloys along the section $\text{Al}_x(\text{Cu}_{50}\text{Si}_{50})_{1-x}$ at 1375 K is shown as the lines with calculated values based on the models. the experimental values of the pre-exponential factors for Al, Cu, and Si were taken from the studies in [77, 79], respectively.

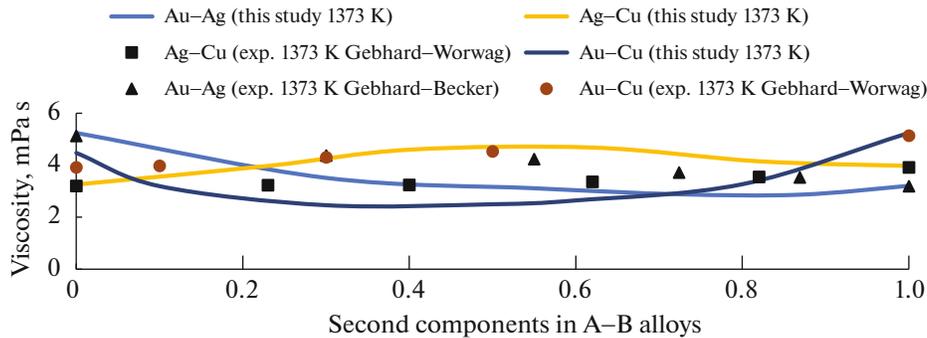


Fig. 9. Calculated viscosities in liquid Au–Ag, Au–Cu, and Ag–Cu alloys at 1373 K. Experimental values of these alloys are taken from [29, 45].

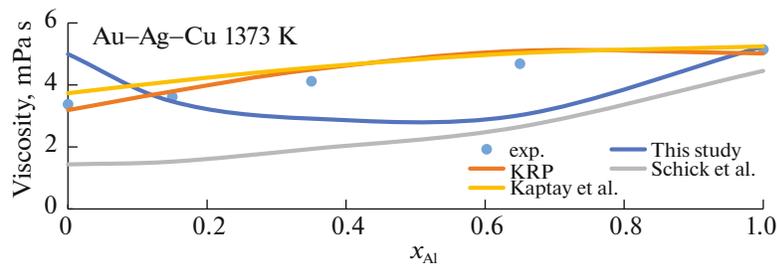


Fig. 10. The dependence of viscosity on composition at 1373 K for section $x_{\text{Ag}}/x_{\text{Cu}} = 0.543$ along with calculated values based on the models. Experimental values of this alloy is taken from [45].

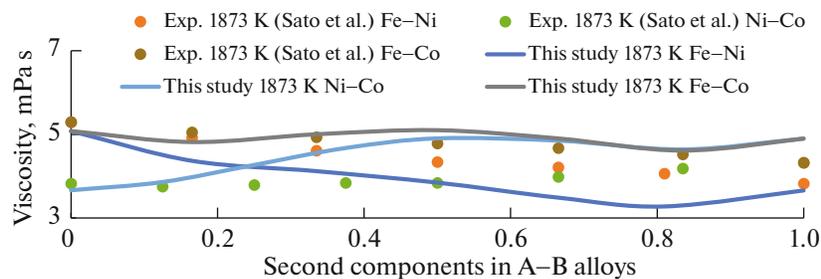


Fig. 11. A comparison between calculated viscosity curves and experimental points for three constitutive binary alloys, Fe–Ni, Fe–Co, and Ni–Co. The experimental values of this alloy is taken from [53].

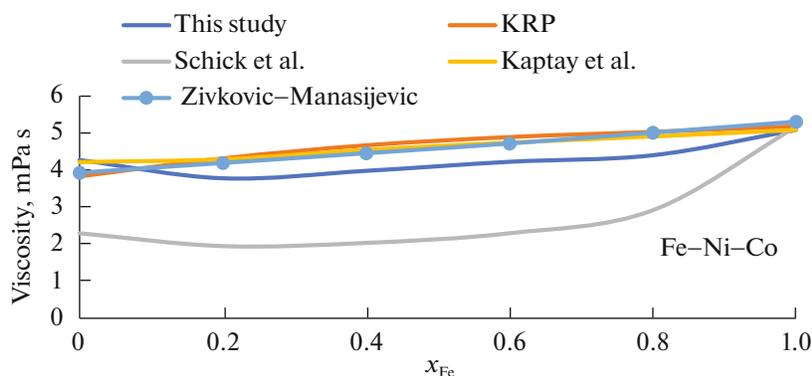


Fig. 12. The dependence of viscosity on composition at 1873 K for section $x_{\text{Ni}}/x_{\text{Co}} = 3/1$ along with calculated values based on the models. The experimental values of the pre-exponential factors for Fe, Ni, and Co were taken from the studies in [79].

For Au–Ag–Cu at 1373 K, the dependences of viscosity on composition Au for all models and experimental results at 1373 K for section $x_{\text{Ag}}/x_{\text{Cu}} = 0.543$ are shown in Fig. 10. The results showed that viscosity increase slightly with increase of Au composition. KRP and Kaptay models except for Schick et al. and these models are in agreement with experiment results.

For Fe–Ni–Co at 1873 K and the section $x_{\text{Ni}}/x_{\text{Co}} = 3/1$, the results showed that viscosity increase slightly with increase of Fe composition. The determined viscosity results show that the viscosity values are in mutually agreement with the results obtained from Chou, Zivkovic-Manasijevic, KRP, and Kaptay models except for Schick et al.

Instead of examining the viscosity curves of the ternary alloys, the statistical analysis for each model was carried out. For this reason, the reproducibility was evaluated by way of the root mean square values of the surface tension of liquid alloy systems in which they were also calculated for the alloys Al–Cu–Si and Au–Ag–Cu at 1375 and 1373 K, respectively. The values in question have been given in Table 6. According to the values in this table, Schick et al. and KRP models which are derived from the physical quantities among the models discussed provide best description of the viscosity for the Al–Cu–Si and Au–Ag–Cu alloys, respectively.

Table 6. The mean square deviation analysis for the liquid Al–Cu–Si and Au–Ag–Cu alloys

Alloys	This study	Schick et al.	KRP	Kaptay
Al–Cu–Si	0.1648	0.0703	0.2194	0.1460
Au–Ag–Cu	0.5229	0.8370	0.1253	0.1664

REFERENCES

1. E. Moelwyn-Hughes, *Physical Chemistry* (Pergamon, Oxford, 1961), p. 798.
2. Z. Morita, T. Iida, and M. Ueda, *Liq. Met.* **1976**, 600 (1977).
3. L. Y. Kozlov, L. Romanov, and N. Petrov, *Izv. Vyssh. Uchebn. Zaved., Chern. Metall.*, No. 3, 7 (1983).
4. M. Kucharski, *Z. Metallkd.* **77**, 393 (1986).
5. M. J. Hirai, *ISIJ. Int.* **33**, 251 (1993).
6. S. D. J. M. Seetharaman and M. T. B. Sichen, *Metall. Mater. Trans. B* **25**, 589 (1994).
7. G. Kaptay, in *Proceedings of MicroCAD 2003 International Conference, 2003*, p. 23.
8. G. Toop, *Trans. TMS-AIME* **223**, 850 (1965).
9. M. Hillert, *Calphad* **4**, 1 (1980).
10. F. Kohler, *Monatsh. Chem.* **91**, 738 (1960).
11. Y. M. Muggianu, M. Gambino, and J. Bros, *J. Chim. Phys. Phys.-Chim. Biol.* **72**, 83 (1975).
12. K.-C. Chou, *Calphad* **19**, 315 (1995).
13. S. Z. Beer, *Liquid Metals: Chemistry and Physics* (Marcel Dekker, New York, 1972).
14. T. Faber, *Introduction to the Theory of Liquid Metals* (Cambridge Univ. Press, London, 1972).
15. Y. Sato, *Trend of Energy Consumption in Residential and Commercial Sector* (The Ministry of Economy, Trade and Industry, Japan, 2004).
16. K. C. Mills, *Recommended Values of Thermophysical Properties for Selected Commercial Alloys* (Woodhead, 2002).
17. D. Wang and R. A. Overfelt, *J. Thermophys.* **23**, 1063 (2002).
18. T. Yamasaki, S. Kanatani, Y. Ogino, et al., *J. Non-Cryst. Solids* **156**, 441 (1993).
19. P. Arsentev and K. Polyakova, *Sov. Non-Ferr. Met. Res.* **5**, 53 (1977).
20. M. Pakiewicz, Ph. D. Dissertation (Syracuse Univ., New York, 1970).
21. E. Gebhardt, M. Becker, and S. Dorner, *Aluminium* **31**, 315 (1955).

22. E. Rothwell, in *Handbook of the Physicochemical Properties of the Elements*, Ed. by G. V. Samsonov (Plenum, Washington, DC, 1961–1962), p. 389.
23. M. Kehr, W. Hoyer, and I. Egry, *J. Thermophys.* **28**, 1017 (2007).
24. J. Brillo, R. Brooks, I. Egry, et al., *Int. J. Mater. Res.* **98**, 457 (2007).
25. H. Schenck, M. G. Frohberg, and K. Hoffmann, *Arch. Eisenhüttenwesen* **34**, 93 (1963).
26. G. Cavalier, in *Proceedings of the Symposium on the Physical Chemistry of Metallic Solutions and Intermetallic Compounds, 1959*, p. 2.
27. R. Barfield and J. Kitchener, *J. Iron Steel Inst.* **180**, 324 (1955).
28. E. Gebhardt, M. Becker, and S. Schäfer, *Z. Metallkd.* **43**, 292 (1952).
29. E. Gebhardt and G. Worwag, *Z. Metallkd.* **42**, 358 (1951).
30. T. Iida, Z.-I. Morita, and S. Takeuchi, *J. Jpn. Inst. Met.* **39**, 1169 (1975).
31. F. Lihl, E. Nachtigall, and A. Schwaiger, *Z. Metallkd.* **59**, 213 (1968).
32. M. Culpin, *Proc. Phys. Soc., Sect. B* **70**, 1079 (1957).
33. E. Gebhardt, M. Becker, and H. Sebastian, *Z. Metallkd.* **46**, 669 (1955).
34. Y. Sato, Y. Kameda, T. Nagasawa, et al., *J. Cryst. Growth* **249**, 404 (2003).
35. S. Nishimura, S. Matsumoto, and K. Terashima, *J. Cryst. Growth* **237**, 1667 (2002).
36. Z. Zhou, S. Mukherjee, and W.-K. Rhim, *J. Cryst. Growth* **257**, 350 (2003).
37. Y. Plevachuk, V. Sklyarchuk, A. Yakymovych, et al., *Metall. Mater. Trans. A* **39**, 3040 (2008).
38. N. Y. Konstantinova and P. Popel, *J. Phys.: Conf. Ser.* **98**, 2008, 062022.
39. M. Schick, J. Brillo, I. Egry, et al., *J. Mater. Sci.* **47**, 8145 (2012).
40. H. A. Friedrichs, L. W. Ronkow, and Y. Zhou, *Steel Res.* **68**, 209 (1997).
41. J. Schmitz, B. Hallstedt, J. Brillo, et al., *J. Mater. Sci.* **47**, 3706 (2012).
42. J. Brillo, I. Egry, and J. Westphal, *Int. J. Mater. Res.* **99**, 162 (2008).
43. M. Adachi, M. Schick, J. Brillo, et al., *J. Mater. Sci.* **45**, 2002 (2010).
44. S. Gruner and W. Hoyer, *J. Alloys Compd.* **460**, 496 (2008).
45. E. Gebhardt and M. Becker, *Z. Metallkd.* **42**, 111 (1951).
46. E. Gebhardt and G. Worwag, *Z. Metallkd.* **43**, 106 (1952).
47. X. Zhong, Y. Liu, K.-C. Chou, et al., *J. Phase Equilib.* **24**, 7 (2003).
48. S. Morioka, *Mater. Sci. Eng.: A* **362**, 223 (2003).
49. D. Živković and D. Manasijević, *Calphad* **29**, 312 (2005).
50. M. G. Frohberg and R. Weber, *Rheol. Acta* **3**, 238 (1964).
51. N. Bodakin, B. Baum, and G. Tyagunov, *Chern Metall.* **7**, 9 (1978).
52. S. Watanabe, *Trans. Jpn. Inst. Met.* **12**, 17 (1971).
53. Y. Sato, K. Sugisawa, D. Aoki, et al., *Meas. Sci. Technol.* **16**, 363 (2005).
54. N. Bodakin, B. Baum, and G. Tyagunov, *Izv. Vyssh. Uchebn. Zaved., Chern. Metall.*, No. 5 18 (1977).
55. A. Adachi, Z. Morita, Y. Ogino, et al., in *Proceedings of the 2nd International Conference on Properties of Liquid Metals, 1973*, p. 561.
56. S. Seetharaman, D. Sichen, and F.-Z. Ji, *Metall. Mater. Trans. B* **31**, 105 (2000).
57. K.-C. Chou and S.-K. Wei, *Trans. Jpn. Inst. Met.* **28**, 439 (1997).
58. Y. Liu, X. Lv, C. Bai, et al., *J. Ind. Eng. Chem.* **30**, 106 (2015).
59. Y. Liu, X. Lv, and C. Bai, *ISIJ Int.* **57**, 1296 (2017).
60. G.-H. Zhang and K.-C. Chou, *Fluid Phase Equilib.* **286**, 28 (2009).
61. G.-H. Zhang and K.-C. Chou, *J. Solut. Chem.* **39**, 1200 (2010).
62. A. Dogan and H. Arslan, *Philos. Mag.* **98**, 37 (2018).
63. A. Dogan and H. Arslan, *Philos. Mag.* **96**, 2887 (2016).
64. A. Dogan and H. Arslan, *Philos. Mag.* **99**, 267 (2019).
65. A. Dogan and H. Arslan, *Philos. Mag.* **96**, 459 (2016).
66. T. Miki, N. Ogawa, T. Nagasaka, et al., *ISIJ Int.* **42**, 1071 (2002).
67. N. Saunders and A. Miodownik, in *Phase Diagrams of Binary Copper Alloys*, Ed. by P. R. Subramanian, D. J. Chakrabarti, and D. E. Laughlin (ASM Int., Materials Park, 1994), p. 412.
68. S. Hassam, J. Ägren, M. Gaune-Escard, et al., *Metall. Trans. A* **21**, 1877 (1990).
69. B. Sundman, S. G. Fries, and W. A. Oates, *CALPHAD* **22**, 335 (1998).
70. A. Kusoffsky, *Acta Mater.* **50**, 5139 (2002).
71. A. Morachevskii, L. S. Tsemekhman, L. Tsymbulov, et al., *Russ. J. Appl. Chem.* **76**, 1728 (2003).
72. V. Witusiewicz, I. Arpshofen, H.-J. Seifert, et al., *Thermochim. Acta* **356**, 39 (2000).
73. K. Fitzner and O. Kleppa, *Metall. Trans. A* **24**, 1827 (1993).
74. W. Krieger, *Berg-Hüttenm. Monatsh.* **122**, 485 (1977).
75. R. Berger, A. Kopp, and H. Philipson, *A Feasibility to Electrify the Combustion Heated Walking Beam Furnace: Applying Induction and Resistance Heating* (KTH Roy. Inst. Technol., Stockholm, Sweden, 2018).
76. J. J. Valencia and P. N. Quested, in *ASM Handbook* (ASM Int., Materials Park, OH, 2008), p. 468.
77. H. Kobatake, J. Schmitz, and J. Brillo, *J. Mater. Sci.* **49**, 3541 (2014).
78. Y. Kawai and Y. Shiraishi, *Handbook of Physico-Chemical Properties at High Temperatures* (Iron and Steel Inst. Japan, 1988).
79. L. Battezzati and A. Greer, *Acta Metall.* **37**, 1791 (1989).
80. A. Crawley, *Int. Metall. Rev.* **19**, 32 (1974).