Device performance enhancement by replacing bulk material with tailor-made metamaterials

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Background

Love waves are traditionally preferred in liquid sensing applications due to the direction of surface motion as shown in Figure 1.

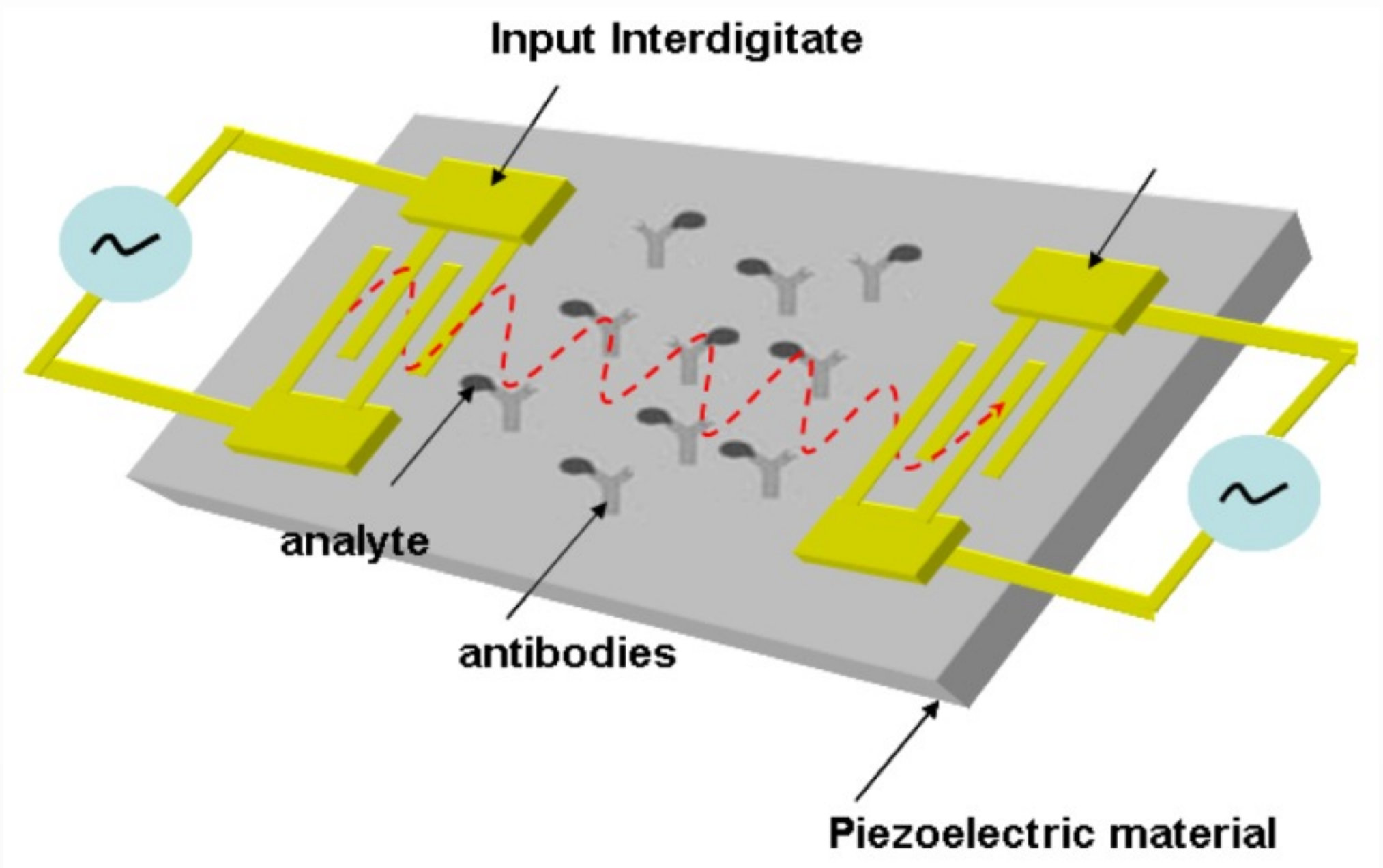

Figure 1: Transverse particle motion in Love waves [1].

Currently, Love wave resonators are exploited in deeptech applications such as particle separation, filtering, wave steering, acoustofluidics, lab-on-chip applications, and alike [2, 3].

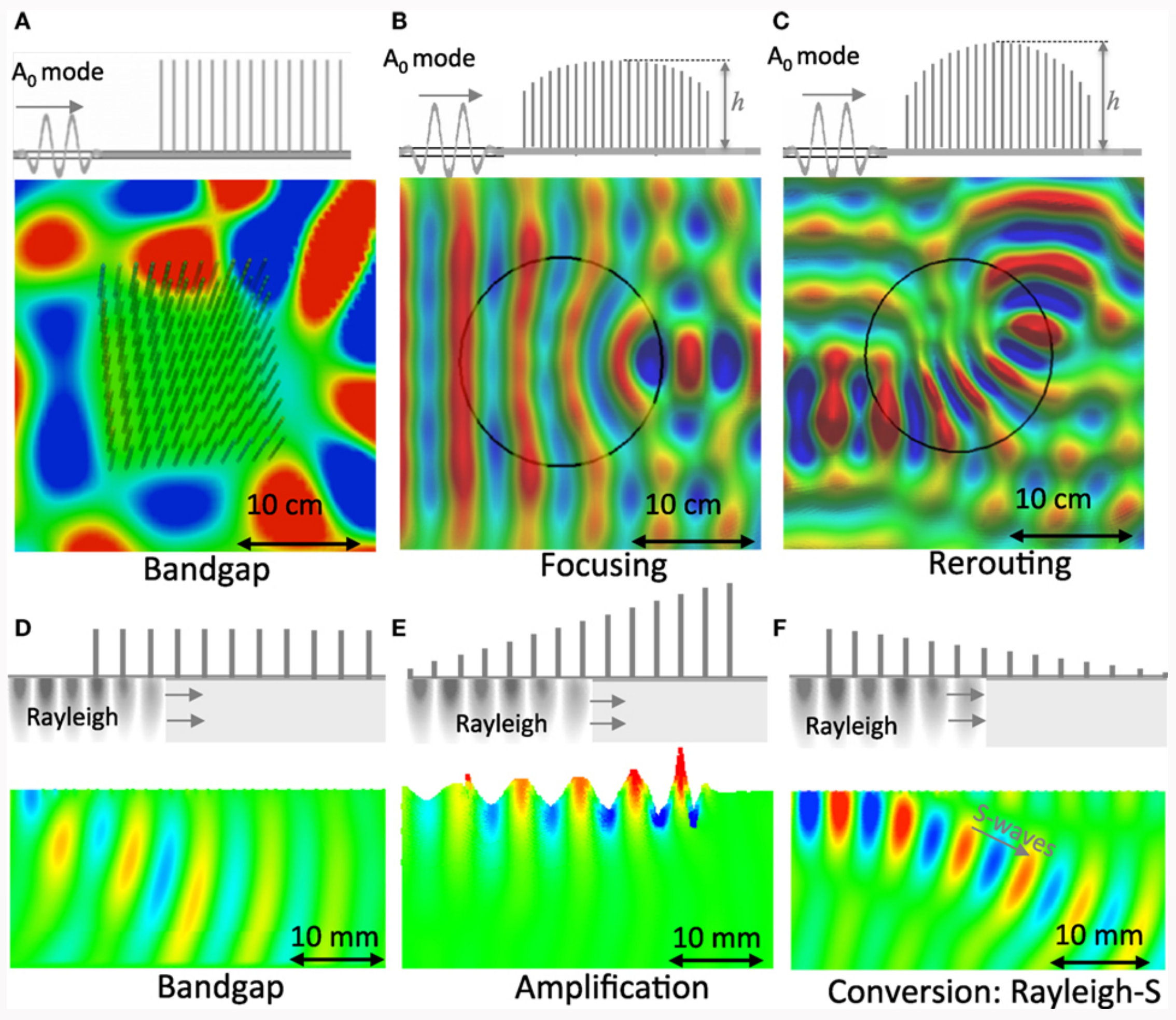

Figure 2: Possible wave guiding applications [4]

Such advancements require materials research in sensor design for performance enhancement by means of novel materials such as auxetic metamaterials, phononic crystals, etc. [5, 6].

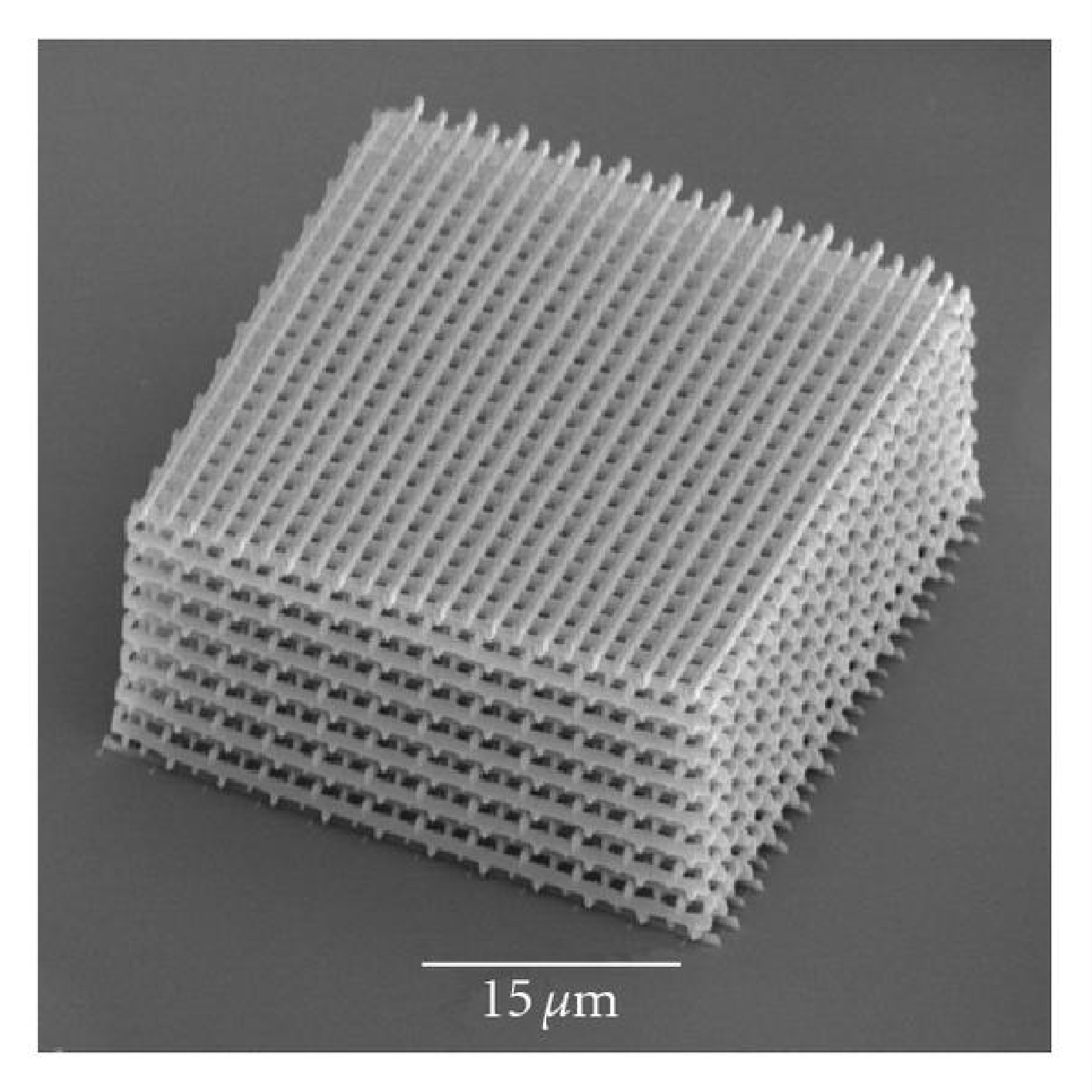

Figure 3: Magnified view of the grid like substructures obtained by 2pp [7]

Based on the results presented in this poster, we intend to extend our work by implementing the Floquet-Bloch theorem to characterize the acoustical properties of auxetic metamaterials produced by 2pp printing.

Modeling

We start by writing the weak form of the mechanical and electrical balance principles combined into the single equation

$$\int_\Omega (\rho\delta u_i \ddot{u}_i + \delta u_{i,j} C_{jikl} S_{kl} - \delta u_{i,j} e_{kji} E_k \\ + \delta\phi_{,i} e_{ikl} S_{kl} + \delta\phi_{,i} \varepsilon_{ik} E_k)\, \mathrm{d}V \; . \tag{1}$$

where displacement, u, and electric potential, ϕ, are simulated by using the material parameters C, e, ε. Strain, S, and electric field, E, are the usual kinematic variables.

Using linear material properties allows the use of a Bernoulli separation ansatz on the unknown fields as follows:

$$u_j(x,t) = u_j(x) \exp(-i\omega t) \; , \\ \phi(x,t) = \phi(x) \exp(-i\omega t) \; ,$$

where ω is the harmonic excitation speed. In this manner, rate terms are rewritten, $\ddot{u}_j = -\omega^2 u_j$. The fields are approximated by using shape functions, N_i and N, as vector and scalar depending only on space,

$$u_i = \sum_{\mathrm{ID}} N_i^{\mathrm{ID}} \hat{u}^{\mathrm{ID}} \; , \; \delta u_i = \sum_{\mathrm{ID}} N_i^{\mathrm{ID}} \mathbf{1}^{\mathrm{ID}} \\ \phi = \sum_{\mathrm{ID}} N^{\mathrm{ID}} \hat{\phi}^{\mathrm{ID}} \; , \; \delta\phi = \sum_{\mathrm{ID}} N^{\mathrm{ID}} \mathbf{1}^{\mathrm{ID}}$$

As we sum up over elements, denoted by $\sum_e$, and obtain $\hat{\boldsymbol{u}}$ and $\hat{\boldsymbol{\phi}}$ as arrays of the nodal values within the computational domain, the first expression within the integrand in Eq. 1 is assembled as the mass matrix,

$$\begin{aligned} \boldsymbol{K}_{uu} \cdot \hat{\boldsymbol{u}} &= \sum_e \int_\Omega \delta u_{j,i} C_{ijkl} S_{kl} \, \mathrm{d}V \\ \boldsymbol{K}_{u\phi} \cdot \hat{\boldsymbol{\phi}} &= -\sum_e \int_\Omega \delta u_{j,i} e_{kij} E_k \, \mathrm{d}V \\ \boldsymbol{K}_{\phi u} \cdot \hat{\boldsymbol{u}} &= \sum_e \int_\Omega \delta\phi_{,i} e_{ikl} S_{kl} \, \mathrm{d}V \\ \boldsymbol{K}_{\phi\phi} \cdot \hat{\boldsymbol{\phi}} &= \sum_e \int_\Omega \delta\phi_{,i} \varepsilon_{ik} E_k \, \mathrm{d}V \\ -\omega^2 \boldsymbol{M} \cdot \hat{\boldsymbol{u}} &= -\sum_e \omega^2 \int_\Omega \rho \delta u_j u_j \, \mathrm{d}V \end{aligned} \tag{2}$$

Eigenvalue Analysis

By observing Eq. $2_{2,3}$, we realize that $\mathbf{K}_{\phi u}$ is the transpose of $\mathbf{K}_{u\phi}$. Now, we rewrite in the matrix form

$$\left(\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\phi} \\ \boldsymbol{K}_{u\phi}^{\mathsf{T}} & \boldsymbol{K}_{\phi\phi} \end{bmatrix} - \omega^2 \begin{bmatrix} \boldsymbol{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right) \begin{Bmatrix} \hat{\boldsymbol{u}} \\ \hat{\boldsymbol{\phi}} \end{Bmatrix} = 0 \; . \tag{3}$$

Solving for $\boldsymbol{\phi}$ from the second line of Eq.3 and substituting into the first line we get:

$$(\boldsymbol{K}_{uu} - \boldsymbol{K}_{u\phi} \cdot \boldsymbol{K}_{\phi\phi}^{-1} \cdot \boldsymbol{K}_{u\phi}^{\mathsf{T}} - \omega^2 \boldsymbol{M}) \cdot \hat{\boldsymbol{u}} = 0 \; .$$

The condensed stiffness matrix,

$$\boldsymbol{K}^* = \boldsymbol{K}_{uu} - \boldsymbol{K}_{u\phi} \cdot \boldsymbol{K}_{\phi\phi}^{-1} \cdot \boldsymbol{K}_{u\phi}^{\mathsf{T}} \; ,$$

enables to rewrite

$$(\boldsymbol{K}^* - \omega^2 \boldsymbol{M}) \hat{\boldsymbol{u}} = 0 \; .$$

The implementation of the model and computations presented in the section below are performed in the open-source finite-element analysis platform Fenics [8].

Simulations

Relevant Love modes were obtained for models having various relative layer thickness values.

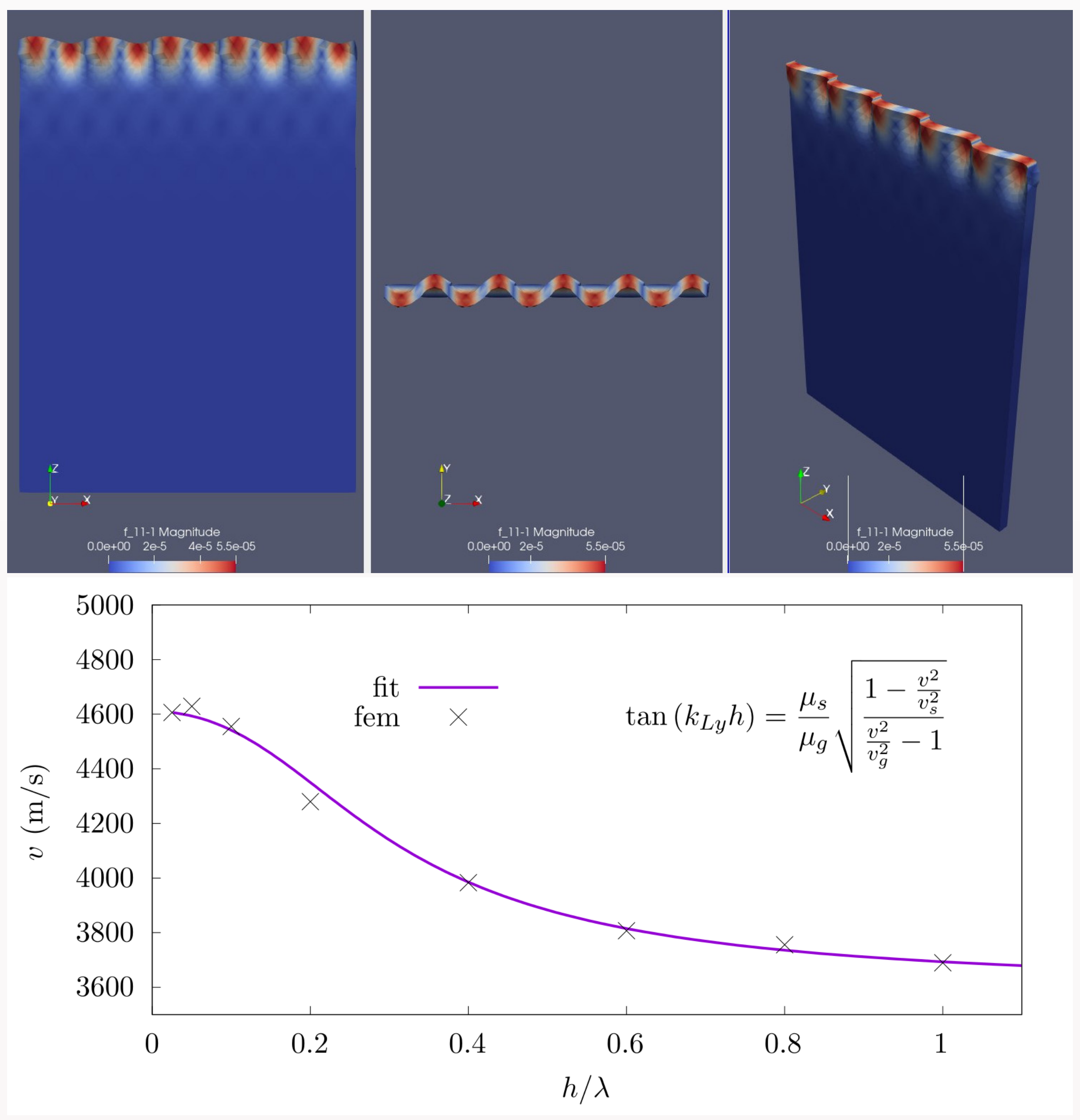

Figure 4: Simulation of the captured Love mode (top), dispersion of wave speed with relative layer thickness (bottom)

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